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Estimating the creep strain to failure of PP at different load levels based on short term tests and Weibull characterization

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Abstract. The short and long term creep behavior is one of the most important properties of polymers used for engineering applications. In order to study this kind of behavior of PP tensile and short term creep measurements were performed and analyzed using long term creep behavior estimating method based on short term tensile and creep tests performed at room temperature, viscoelastic behavior, and variable transformations. Applying Weibull distribution based approximations for the measured curves predictions for the creep strain to failure depending on the creep load were determined and the parameters were found by fitting the measurements. The upper, mean, and lower estimations as well as the confidence interval for the means give a possibility for designers' calculations at arbitrary creep load levels.

Keywords: material testing, short term creep, long term behavior, creep strain to failure, Weibull distribution

1. Introduction

The long term creep behavior of polymers or polymer composites subjected to mechanical load is one of the most important properties considering the fact that all machine parts or other constructions should have a sufficient life span of some years therefore it has been studied for a long time in order to find suitable methods for estimating the expected lifetime at different load levels.

Supposing the polymeric material is of linear viscoelastic nature and/or exhibits simple thermo-rheological behavior the time-temperature superposition principle can be used to construct the long term creep curve called master-curve from short term creep measurements performed at different temperatures and a given load level [1–5]. In general the Williams-Landel-Ferry (WLF) or the Arrhenius equation or some combination of them is applied to

determining the shift factor necessary for constructing the long term master curve [6–10]. Power law type approximations like Nutting's or Findley's ones [6, 8, 11] or simple rheological models such as Standard Solid or Burgers or their generalized forms [1–5, 12] can give a simple solution to extrapolate the measured data or to describe the master curve mathematically. In some cases constitutive mechanical models are developed using measured material constants and numerical analysis [13] or finite element models are created to describe thermoelastic creep [14]. In opposite to them models based on the kinetic, thermal activation, or micro-deformation behavior of molecule chains are developed to predict the creep of polymers [15–18].

In general these methods can be used just at or below a given load level and in most cases they can-not estimate the expected lifetime and/or the creep strain-to-failure.

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Nagy and Vas [19, 20] have developed a combined method for estimating the long term behavior from short term measurements based on the linear viscoelastic theory and non-linear variable transformations and they employed that to stress relaxation. In the present paper this method is applied to estimate the creep strain to failure of PP at arbitrary load levels by making use of the information provided by tensile tests and short term creep measurements performed at room temperature as well as a Weibull distribution based stochastic model.

2. Theoretical considerations

Considering mechanical tests a polymer specimen behaves as a system characterized by operator S that gives a relationship between the time-dependent stimulus as input, $X(t)$, and the response, $Y(t)$, as output (Equation (1)) [19, 20]:

$$Y(t) = S(X)(t) \tag{1}$$

If X_1 and X_2 are two stimuli and X_2 is the integral of X_1 , then the following relations are true (Equation (2)):

$$X_2(t) = I(X_1)(t) = \int_0^t X_1(u)du \leftrightarrow \leftrightarrow X_1(t) = D(X_2)(t) = \frac{dX_2(t)}{dt} \tag{2}$$

where I and D are integral and differential operators, respectively. Supposing the polymer material is of linear viscoelastic (LVE) behavior then operator S is linear therefore in most cases the relations according to Equation (2) are valid for the responses as well (Equation (3)):

$$Y_2(t) = S(X_2)(t) = S(I(X_1))(t) = = I(S(X_1))(t) = I(Y_1)(t) \leftrightarrow \leftrightarrow (Y_1)(t) = D(Y_2)(t) \tag{3}$$

For example, if $X_1(t) = F_1(t) = F_0I(t)$ is a step-function and $X_2(t) = F_2(t) = \dot{F}_0tI(t)$ is a ramp-function where $I(t)$ is the unit jump function, F_0 is the creep load, and \dot{F}_0 is the load rate, hence the responses of the real material are the creep curve, $Y_1(t) = \epsilon_1(t)$, and the tensile test curve, that is the tensile load strain-time relationship recorded, $Y_2(t) = \epsilon_2(t)$. In case of LVE behavior the LVE estimation of the creep curve is the derivative of the tensile test one (Equation (4)) [19, 20]:

$$\epsilon_{L1}(t) = D(\epsilon_2)(t) = \frac{d\epsilon_2(t)}{dt} \tag{4}$$

The creep stimulus above, $F_1(t)$, is of ideal form because it has got a jump of infinite steepness. In the reality the slope should be finite therefore the real creep stimulus is given by Equation (5):

$$F_1(t) = \begin{cases} \dot{F}_0tI(t), & t < t_0 \\ \dot{F}_0t_0, & t \geq t_0 \end{cases} \tag{5}$$

where t_0 is the uploading time needed for setting the creep load level ($F_0 = \dot{F}_0t_0$) (Figure 1a).

Nagy and Vas showed that in the case of real creep stimulus where the uploading, that is the setting the creep load, is performed with constant load rate and assuming LVE behavior the proper form of Equation (4) is as follows (Equation (6)) [19, 20]:

$$\epsilon_{L1}(t) = \epsilon_2(t) - \epsilon_2(t - t_0), \quad t_0 \leq t \leq t_{2B} \tag{6}$$

where $\epsilon_{L1}(t)$ is the LVE response to the stimulus (5) while $\epsilon_2(t)$ is the tensile test curve, t_{2B} is the break-

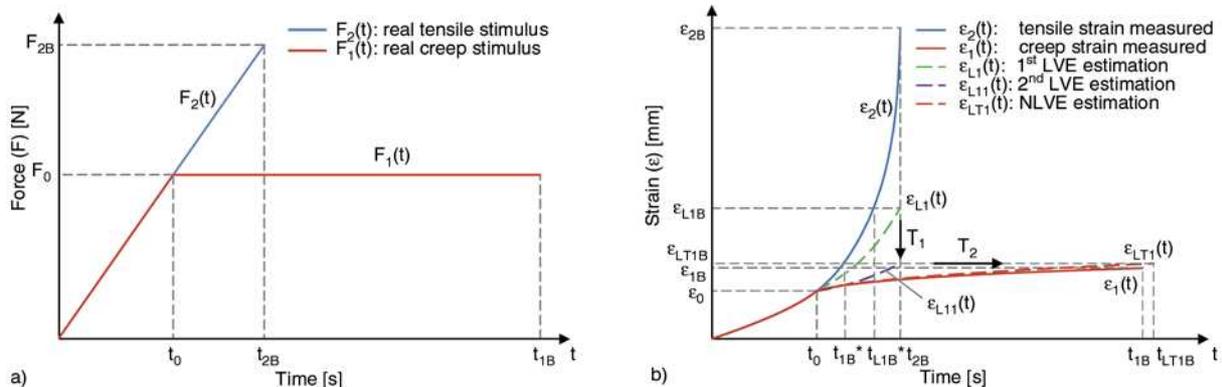


Figure 1. Stimuli of real creep and tensile tests (a) and the LVE estimation of the real creep curve (b)

ing time at tensile testing, consequently $\varepsilon_{2B} = \varepsilon_2(t_{2B})$ is the breaking strain, and $\varepsilon_2(t) = 0$ if $t \leq 0$. Equation (6) can be used if $\varepsilon_2(t)$ is the constant load rate tensile test response of the real polymer material and $\varepsilon_{L1}(t)$ can be considered as a linear viscoelastic (LVE) estimation of the real creep curve (Figure 1b). Real polymers exhibit LVE behavior only in a relatively small load range and beyond it they behave in a non-linear manner. It was also proven by Nagy and Vas [19, 20] that at a given creep load the real creep curve can be estimated by a non-linear variable transformation (T) of the LVE estimation that has got a kind of analogy with the load level – time superposition principle described by Urzumtsev and Maksimov [5] similar to that used in the case of the time-temperature superposition principle (Equation (7)):

$$\varepsilon_1(t) \approx T_1(\varepsilon_{L1}(T_2(t))) =: \varepsilon_{LT1}(t) \quad (7)$$

where T_1 and T_2 are the component transformations of T (Figure 1b). T_2 is determined to be non-linear, however, T_1 can be realized in a linear form as follows (Equation (8)):

$$\begin{aligned} \varepsilon_{L11}(t) = T_1(\varepsilon_{L1}(t)) &= \varepsilon_{L1}(t_0) + c(\varepsilon_{L1}(t) - \varepsilon_{L1}(t_0)) = \\ &= \varepsilon_0 + c(\varepsilon_{L1}(t) - \varepsilon_0) \end{aligned} \quad (8)$$

where ‘c’ is a transformation constant and ε_0 is the creep strain load determined by the uploading time and defined by Equation (9):

$$\varepsilon_0 = \varepsilon_{L1}(t_0) = \varepsilon_2(t_0) \quad (9)$$

Let t_{1B} be the real lifetime at creep hence the creep strain to failure is $\varepsilon_{1B} = \varepsilon_1(t_{1B})$ (Equation (10)) which can be estimated by Equation (8):

$$\begin{aligned} \varepsilon_{1B} \approx \varepsilon_{L11}(t_{2B}) &= T_1(\varepsilon_{L1}(t_{2B})) = \\ &= \varepsilon_0 + c(\varepsilon_{L1}(t_{2B}) - \varepsilon_0) \end{aligned} \quad (10)$$

It is obvious that the primary LVE estimation of the creep strain to failure is $\varepsilon_{L1}(t_{2B})$.

The steps of the whole calculation process can be summarized as follows (see Figure 1b).

1. Tensile test measurements ($\varepsilon_{2i}(t)$, $i=1, \dots, n$) \rightarrow
2. Mean tensile test curve ($\varepsilon_2(t)$) averaging by point by point \rightarrow
3. Primary LVE estimation of the creep

process ($\varepsilon_{L1}(t)$) according to Equation (6) \rightarrow 4. Secondary LVE estimation of the creep process ($\varepsilon_{L11}(t)$) by linear transformation T_1 according to Equations (7) and (8) using the mean values of the creep strain to failure measured at higher creep load levels which makes it possible to estimate the mean creep strain to failure \rightarrow 5. NLVE estimation of the whole creep process by time transformation T_2 according to Equation (7) up to the creep failure using the short term creep measurements on the basis of which, in the knowledge of the mean creep strain to failure, the main creep time to failure can be estimated.

Steps 1.–3. are preparatory operations, in Step 4 we obtain estimation for the creep strain to failure while Step 5 provides the assessment of the measurable creep curve including the creep failure point with co-ordinates creep strain to failure estimated in Step 4 and creep lifetime as a new result.

In this paper the examinations are confined to steps 1–4 and it will be shown that in the case of PP tested Equations (8) and (10) can be used for estimating the creep strain to failure at arbitrary creep load level, as well as the right side of Equation (10) can be described by a stochastic model based on the Weibull distribution that will make it easier to determine the parameters of the tensile-testtime \rightarrow creep-test-time transformation (T_2) in Step 5 planned to be carried out in a later paper.

3. Material and test methods

An isotactic polypropylene homopolymer (Tipplen H 949A from TVK, Tiszaújváros, Hungary) was applied to constant force rate tensile tests and creep measurements. The material was regranulated with Brabender Plastograph extruder machine to achieve a similar thermal prehistory as the glass fiber reinforced PP composites to be examined later [21].

148 mm long type 1A dumbbell specimens of 10 mm width and 4 mm thickness according to the ISO 527-2 Standard were injection molded on an Arburg Allrounder 320 C-GE 500-170 injection molding machine (Arburg, Germany).

For the tests a Zwick Z-005 universal testing machine (with 5 kN nominal capacity standard load cell) was used in constant force rate mode, where the constant force rate was 50 N/s until the specimen broke in tensile tests or the preset load level was reached in creep tests. The gauge length was 100 mm

long hence the elongation in millimeter equals the strain in percent numerically. In the creep tests after the uploading the load was held constant for 10 hours (or until the specimen failed) and the length variation of the specimen was measured using the crosshead signal. The applied load levels were determined in 10 percent steps of the measured average breaking force. Each test was carried out under uniaxial tensile load and at room temperature ($23\pm 1^\circ\text{C}$).

4. Measurements

Thirty constant force rate tensile tests were performed on injection molded dumbbell specimens. At the starting phase of uploading due to the inertia of the engine system of the universal testing machine and the time-delay of the data collection a zero-point error can be observed which could cause serious difficulties in calculating the correct mean curves. In order to obtain correct averaged characteristics the curves were shifted one by one as a function of time till the initial tangents of the measured strain-time and force-time curves crossed

exactly the origin (Figure 2a). After shifting the curves several starting points were corrected to fit into the initial tangent (Figure 2b).

The corrected results of the tensile measurements were averaged. The averaged tensile curve showed indeterminate strain values not fitted to the mean of the others in the breaking section (Figure 3a). This had to be corrected and smoothed by shifting the end section of the strain-time curve so they could be used for later calculations (Figure 3b). According to the 30 tensile measurements the average breaking strain was 10.69% with a standard deviation of 0.728% and the average breaking force of the measurements was 1610 ± 19.6 N.

On the other hand five creep measurements were performed at every load level that were 10, 20, 30, 40, 50, 60, 65, 70, 75, 80, 85, 90 and 95% of the mean breaking force taken at 1600 N.

Each of the altogether 65 creep tests lasted 10 hours long or till the specimen was failed. The curves were averaged point by point in the way described above and if it was necessary (where the specimen was broken within the creep measurement) smoothed.

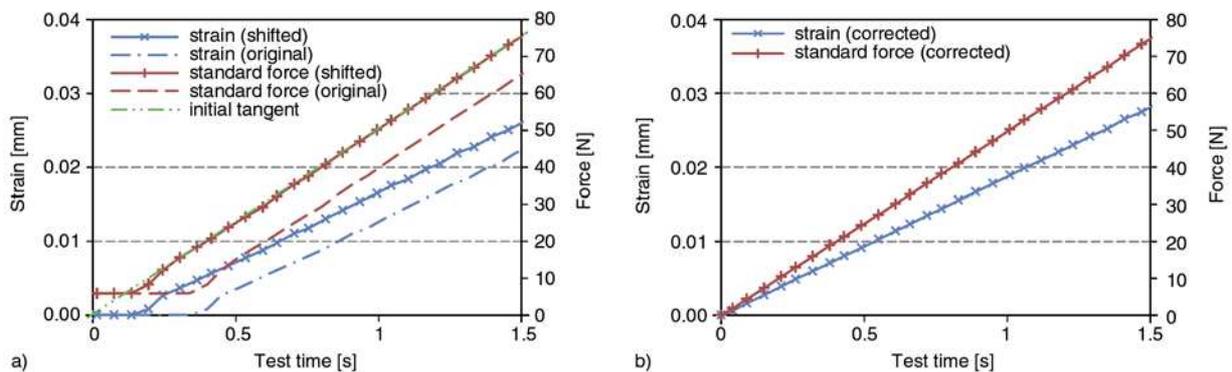


Figure 2. Shifting the tensile curves to the origin using an initial tangent (a) and tensile curves with corrected starting points (b)

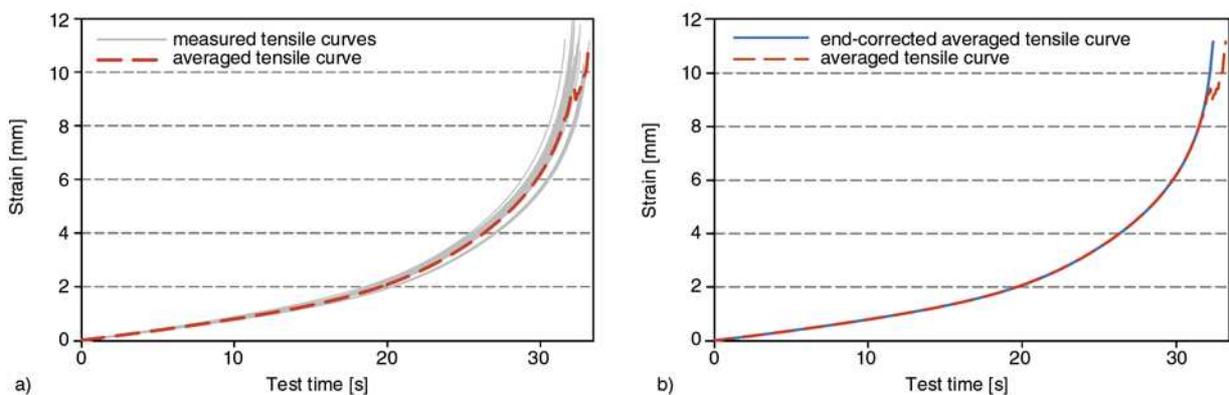


Figure 3. Averaging the tensile curve bundle (a) and smoothing the indeterminate strain values at the breaking range of shifted curves (b)

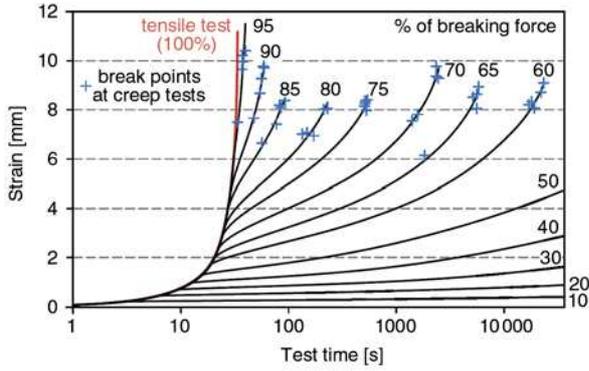


Figure 4. Creep results carried out at different tensile load levels

The averaged and smoothed creep curves can be seen in Figure 4 where all the creep failure points observed are depicted as well.

5. Evaluation and discussion

5.1. Estimation of LVE creep strain to failure based on Weibull distribution

It is obvious that the values of the LVE creep strain to failure, $\epsilon_{L1B}(t_0)$, determined from the real tensile test curve, $\epsilon_2(t)$, are dependent on creep load level, F_0 , or the uploading time, t_0 , that is proportional to F_0 , and create a monotonically increasing and convergent sequence with increasing t_0 or F_0 like the measured ones according to Ehrenstein (Figure 88 in page 115 of [2]) therefore there exists a finite value ($0 < \epsilon_{L1B\infty} < \infty$) as least upper bound. The creep strain to failure values normalized by this upper bound ($\epsilon_{L1B\infty}$) fall in interval $[0,1]$ and take up value 1 at a certain loading time ($0 \leq t_{0\infty} \leq \infty$).

All this means that these normalized failure strain values meet the formal requirements of a probability distribution function. On the other hand, this function can be understood as a distribution characteristic of a strength variable, σ_{B0} , because the values of the independent variable are time values proportional to the load and the dependent variable is a ratio connected to failure events that expresses how close the failure is to the stress $\sigma_{2B} = F_{2B}/A_0$ representing the sure and sudden breakage or fracture (A_0 is the cross section of the specimens before loading and here $A_0 = 40 \text{ mm}^2$). This strength variable, σ_{B0} , is in relation to the real creep strain to failure (ϵ_{1B}) that can be detailed in the following way. Using the measured creep curve, $\epsilon_1(t)$, the creep strain to failure, ϵ_{1B} , belongs to the creep failure time (t_{1B}), that

is the creep lifetime, t_{1B} : $\epsilon_{1B} = \epsilon_1(t_{1B})$. The first LVE estimation of ϵ_{1B} is $\epsilon_{L1B} = \epsilon_{L1}(t_{2B})$ (Figure 1b). Considering the intersection point of the tensile test curve, $\epsilon_2(t)$, and the creep strain to failure level (ϵ_{1B}) (see Figure 1b), let its projection onto the tensile time axis be denoted by $t^*_{1B} = \epsilon_2^{-1}(\epsilon_{1B}) = \epsilon_2^{-1}(\epsilon_1(t_{1B}))$ and a first LVE estimation of which can be $t^*_{L1B} = \epsilon_2^{-1}(\epsilon_{L1B})$. Because constant load rate tensile test was used ($\dot{F}_0 = 50 \text{ N/s}$) the stress corresponding to t^*_{L1B} is $\sigma^*_{L1B} = \dot{\sigma}_0 t^*_{L1B}$ where $\dot{\sigma}_0 = \dot{F}_0/A_0$. The distribution function of σ^*_{L1B} is equal to those of σ_{B0} and a kind of transformed creep lifetime variable, $\tau = \sigma_{B0}/\dot{\sigma}_0$, is in strict relation with that of the first LVE estimation of the creep strain to failure (ϵ_{L1B}). Taking into account that the statistical properties related to failure such as stress-, strain-, and time-to-failure have got a kind of minimum nature therefore they can be described with the Weibull distribution that is the extreme value distribution of minima [22] under sufficiently general conditions in both practice and theory [23] shown by e.g. Phoenix [24, 25], Wagner *et al.* [16], or later Raghavan and Meshii [17], Vujosevic and Krajcinovic [18], or recently Fancey [26]. Hence it can be applied to the distribution of both the strength, σ_{B0} , and the transformed lifetime, τ , as well (Equation (11)). All that can be followed well in Equation (11):

$$\begin{aligned}
 P\left(\sigma^*_{L1B} = \dot{\sigma}_0 t^*_{L1B} < \sigma_2 = \dot{\sigma}_0 t_2 = \dot{\sigma}_0 \frac{t_2}{t_0} t_0\right) &= \\
 &= P(\sigma_{B0} = \alpha_{20} \sigma^*_{L1B} < \sigma_0 = \dot{\sigma}_0 t_0) = \\
 &= P\left(\tau = \frac{\sigma_{B0}}{\dot{\sigma}_0} = \alpha_{20} t^*_{L1B} = \alpha_{20} \epsilon_2^{-1}(\epsilon_{L1B}) < t_0\right) = \\
 &= P\left(\epsilon_{L1B} = \epsilon_2\left(\frac{\tau}{\alpha_{20}}\right) < \epsilon_2\left(\frac{t_0}{\alpha_{20}}\right)\right) = \\
 &= 1 - e^{-\left(\frac{t_0}{a}\right)^k} = \frac{\epsilon_{L1B}(t_0)}{\epsilon_{L1B\infty}} \tag{11}
 \end{aligned}$$

where where σ_2 and t_2 are respectively tensile load and tensile time values while $0 < \alpha_{20} = t_0/t_2 < 1$ is a kind of shift factor [4, 5], and $\tau = \alpha_{20} t^*_{L1B} \approx \alpha_{20} t^*_{1B} = \alpha_{20} \epsilon_2^{-1}(\epsilon_1(t_{1B}))$ is the first LVE estimation of a sort of creep lifetime transformed to the tensile time range but it is not the creep time failure, as well as ‘ a ’ and ‘ k ’ are the Weibull scale and modulus parameters, respectively.

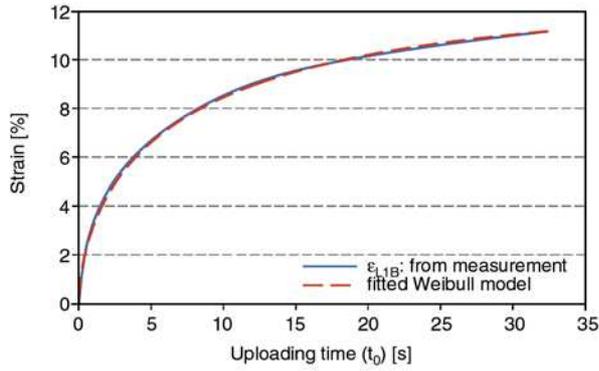


Figure 5. LVE estimation of the creep strain to failure calculated from the tensile test measurements and the Weibull distribution based model fitted

Rearranging Equation (11) yields the relationship between the mean LVE creep strain to failure and uploading time (Equation (12)):

$$\varepsilon_{L1B}(t_0) = \varepsilon_{L1B\infty} \left(1 - e^{-\left(\frac{t_0}{a}\right)^k} \right), 0 \leq t_0 \leq t_{2B} \quad (12)$$

If $t_0 = 0$ then $\varepsilon_{L1B}(0) = 0$, and $t_0 = t_{2B}$ leads to $\varepsilon_{L1B}(t_{2B}) = \varepsilon_{2B}$. According to the measurements the mean values of the latter determined by the endpoint of the averaged tensile test curve are $t_{2B} = 32.37$ s and $\varepsilon_{2B} = 11.17\%$.

Equation (6) at $t = t_{2B}$ gives a possibility to determine $\varepsilon_{L1B}(t_0)$ from the measured tensile test curve, $\varepsilon_2(t)$, because $\varepsilon_{L1B}(t_0) = \varepsilon_{L1}(t_{2B})$ (Figure 5).

Figure 5 shows the LVE estimation of the creep strain to failure as a function of the uploading time determined from the measured tensile test curve and its Weibull distribution based approximation fitted by using Equation (12).

The goodness of this fitting is characterized by the high determination coefficient ($R^2 = 0.995$) (mean squared error in strain is 0.075%) where the parameters a , k , and $\varepsilon_{L1B\infty}$ came about 8.076 s, 0.5875, and 12.47% respectively.

5.2. Weibull distribution based approximation of the tensile test curve

Rearranging Equation (6) gives a recursive formula (Equation (13)) for the tensile test curve:

$$\varepsilon_2(t) = \varepsilon_{L1}(t) + \varepsilon_2(t - t_0), t_0 \leq t \leq t_{2B} \quad (13)$$

This is also true for $t = t_{2B}$ and since $\varepsilon_2(t_{2B}) = \varepsilon_{2B}$ and $\varepsilon_{L1B}(t_0) = \varepsilon_{L1}(t_{2B})$ hence from Equation (13) we get Equation (14):

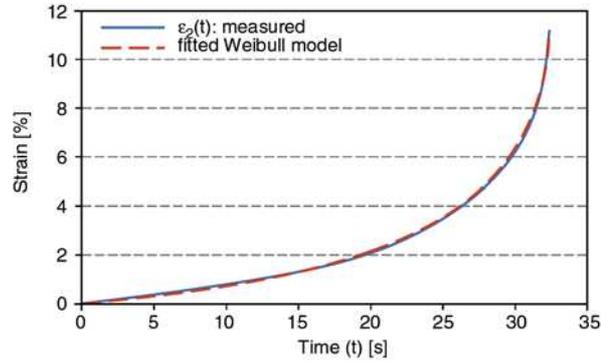


Figure 6. Averaged tensile test curve and its Weibull distribution based approximation

$$\varepsilon_2(t_{2B} - t_0) = \varepsilon_{2B} - \varepsilon_{L1B}(t_0) \quad (14)$$

where the domain of the function on the left side is $[0, t_{2B}]$ therefore its independent variable can be substituted by $t_0 = t_{2B} - t$, $0 \leq t \leq t_{2B}$ (Equation (15)):

$$\varepsilon_2(t) = \varepsilon_{2B} - \varepsilon_{L1B}(t_{2B} - t) \quad (15)$$

Taking Equation (12) at $t_0 = t_{2B} - t$ and substituting it into Equation (15) provides a Weibull distribution based formula by Equation (16) for approximating the mean tensile test curve as an LVE model (Figure 6; mean squared error of the approximation measured in strain is 0.075%).

$$\varepsilon_2(t) \approx \varepsilon_{L2}(t) = \varepsilon_{2B} - \varepsilon_{L1B\infty} \left(1 - e^{-\left(\frac{t_{2B} - t}{a}\right)^k} \right), \quad 0 \leq t \leq t_{2B} \quad (16)$$

Equation (16) gives the breaking strain, ε_{2B} , at setting $t = t_{2B}$ and 0 at $t = 0$.

5.3. Relation between creep strain to failure increment and load level

Subtracting the strain loads, ε_0 , from values $\varepsilon_{L1B}(t_0)$ and plotting them as a function of the uploading time, t_0 , results in a nearly symmetric curve having a maximum that can be described by using the Weibull-distribution based method as well (Equation (17)) (Figure 7).

$$\begin{aligned} \varepsilon_{L1B}(t_0) - \varepsilon_0 &= \varepsilon_{L1B}(t_0) - \varepsilon_2(t_0) = \\ &= \varepsilon_{L1B\infty} \left(1 - e^{-\left(\frac{t_0}{a}\right)^k} + e^{-\left(\frac{t_{2B}}{a}\right)^k} - e^{-\left(\frac{t_{2B} - t_0}{a}\right)^k} \right), \quad 0 \leq t_0 \leq t_{2B} \end{aligned} \quad (17)$$

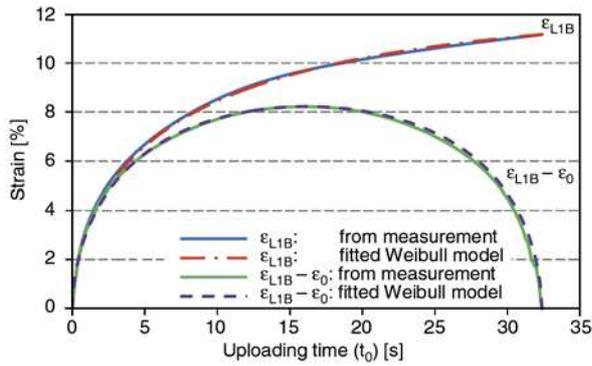


Figure 7. LVE estimation of the creep strain to failure and its increment versus the uploading time (t_0) calculated from tensile measurements and their Weibull distribution based approximation

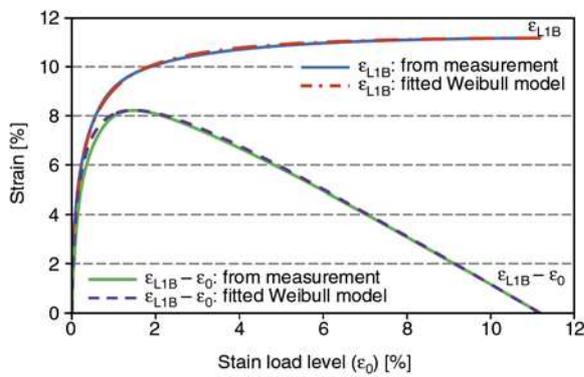


Figure 8. LVE estimation of the creep strain to failure and its increment versus the creep strain load (ε_0) calculated from tensile measurements and their Weibull distribution based approximation

Depicting the curves in Figure 7 as a function of the creep strain load, ε_0 , makes the originally nearly symmetric one strongly asymmetric (Figure 8). The creep strain to failure increment has got a maximum in both cases ($\varepsilon_{L1B} - \varepsilon_0 = 8.23\%$ at $t_0 = 16.2$ s or $\varepsilon_0 = 1.47\%$). The strain value and its place may be characteristic for the material tested therefore it is worth studying the possible changes in structure related to it. The mean squared error of the approximation measured in strain is 0.13%.

5.4. Estimation of the measured creep strain to failure

Using Equation (6) and the tensile test curve modeled with Equation (16) the LVE estimation of the creep curve can be obtained ($t_0 \leq t \leq t_{2B}$) (Equation (18)):

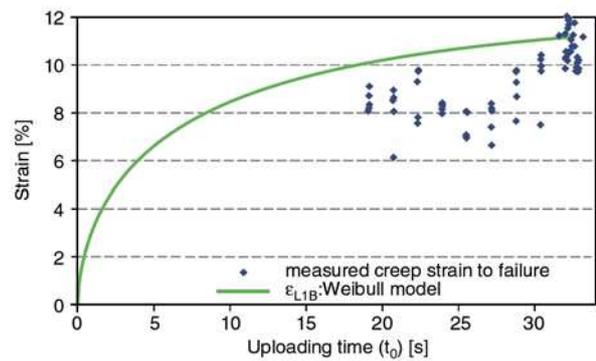


Figure 9. Weibull distribution based LVE estimation and the measured values of the creep strain to failure versus the uploading time

$$\begin{aligned} \varepsilon_{L1}(t) &= \varepsilon_{L2}(t) - \varepsilon_{L2}(t - t_0) = \\ &= \varepsilon_{L1B\infty} \left(e^{-\left(\frac{t_{2B} - t}{a}\right)^k} - e^{-\left(\frac{t_{2B} - t + t_0}{a}\right)^k} \right), \\ & \quad t_0 \leq t \leq t_{2B} \end{aligned} \quad (18)$$

If t_{2B} is the mean value of the fracture times obtained by tensile measurements then $\varepsilon_{L1B}(t_0) = \varepsilon_{L1}(t_{2B})$ is the LVE estimation of the real creep strain to failure, $\varepsilon_{1B}(t_0)$, belonging to uploading time t_0 . On the basis of the creep measurements done at higher strain load levels these $\varepsilon_{L1B}(t_0)$ values can be considered as an upper estimation of the measured ones (Equation (19)) (Figure 9):

$$\varepsilon_{1B}(t_0) \leq \varepsilon_{L1B}(t_0) = \varepsilon_{L1}(t_{2B}), \quad 0 \leq t_0 \leq t_{2B} \quad (19)$$

Applying variable transformation T_1 along the strain axis to Equation (18) provides the first non-linear viscoelastic approximation (NLVE) of the real creep curve ($\varepsilon_{L11}(t)$). Numerical examinations showed that in this case transformation T_1 could be realized as a linear combination using a permanent transformation parameter, $0 < c$, as follows (Equation (20)):

$$\begin{aligned} \varepsilon_{L11}(t) &= \varepsilon_{L1}(t_0) + c(\varepsilon_{L1}(t) - \varepsilon_{L1}(t_0)) = \\ &= (1 - c)\varepsilon_0 + c\varepsilon_{L1B\infty} \left(e^{-\left(\frac{t_{2B} - t}{a}\right)^k} - e^{-\left(\frac{t_{2B} - t + t_0}{a}\right)^k} \right) \end{aligned} \quad (20)$$

The value of this new approximation of the creep curve set at t_{2B} estimates the real creep strain to fail-

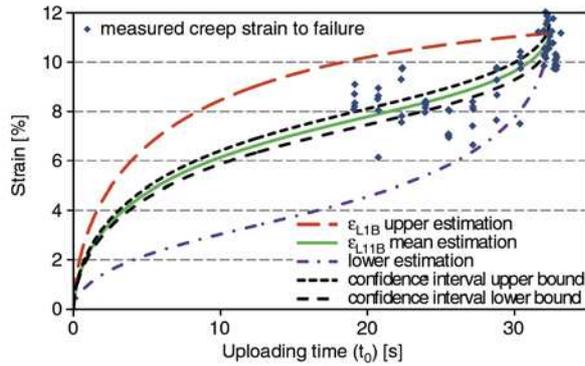


Figure 10. Measured values of the creep strain to failure versus the uploading time and their upper, mean, and lower estimations as well as confidence interval curves for the expected values

ure, $\varepsilon_{1B} = \varepsilon_1(t_{1B})$, observed at instant t_{1B} of the creep failure in a sense of average (Equation (21)):

$$\begin{aligned} \varepsilon_{1B}(t_0) &\approx \varepsilon_{L1B}(t_0) = \varepsilon_{L1}(t_{2B}) = \\ &= (1 - c)\varepsilon_0 + c\varepsilon_{L1B\infty} \left(1 - e^{-\left(\frac{t_0}{a}\right)^k} \right) \end{aligned} \quad (21)$$

Fitting the measured values of the creep strain to failure by using the least square method in order to estimate the mean values the transformation parameter, c , came about 0.7 (Figure 10) (mean squared error in strain was 0.91% indicating a rather large standard deviation).

According to numerical analysis the measured creep strain to failure can be estimated by the LVE approximation from above and its transformed forms calculated with $c = 0.7$ and 0.3 give estimations for the mean and lower values respectively (Figure 10). Besides the mean values the upper and lower estimations for single values provide a range useful for designers’ calculations at arbitrary creep load levels by assessing the probability levels belonging to them. Similarly confidence interval can be constructed for the expected strain to failure values. Let ε_{2B} and $s_{\varepsilon_{2B}}$ be the mean value and the standard deviation of the breaking strain measured by constant load rate tensile tests and calculated from ‘ n ’ measurements. The confidence interval with significance level, $0 < \alpha < 1$, is given for the mean value by Equation (22):

$$\begin{aligned} &\left(\varepsilon_{2B} - t_{\alpha/2,f} \frac{s_{\varepsilon_{2B}}}{\sqrt{n}}; \varepsilon_{2B} + t_{1-\alpha/2,f} \frac{s_{\varepsilon_{2B}}}{\sqrt{n}} \right) = \\ &= \left(\varepsilon_{2B} \left[1 - t_{\alpha/2,f} \frac{V_{\varepsilon_{2B}}}{\sqrt{n}} \right]; \varepsilon_{2B} \left[1 + t_{\alpha/2,f} \frac{V_{\varepsilon_{2B}}}{\sqrt{n}} \right] \right) = \\ &= (\varepsilon_{2B}[1 - \delta_n]; \varepsilon_{2B}[1 + \delta_n]); \delta_n = t_{\alpha/2,f} \frac{s_{\varepsilon_{2B}}}{\sqrt{n}} \end{aligned} \quad (22)$$

where $f = n - 1$ is the degree of freedom and $t_{\alpha/2,f}$ is the two sided confidence coefficient taken from the Student table while $V_{\varepsilon_{2B}} = s_{\varepsilon_{2B}}/\varepsilon_{2B}$ is the relative standard deviation [27]. This can be extended for the mean creep strain to failure values using the method leading from Equation (12) to Equation (21). For simplicity assume that $V_{\varepsilon_{2B}}$ is constant a confidence interval range created by upper (U) and lower (L) borderlines can be estimated for the expected creep strain to failure values at arbitrary creep load levels by multiplying Equation (12) by $(1 \pm \delta_n)$ and transforming that according to Equation (21).

$$\begin{aligned} \varepsilon_{1B}^{U,L}(t_0) &\approx \varepsilon_{L1B}^{U,L}(t_0) = \\ &= (1 - c)\varepsilon_0 + c\varepsilon_{L1B\infty}(1 \pm \delta_n) \left(1 - e^{-\left(\frac{t_0}{a}\right)^k} \right) \end{aligned} \quad (23)$$

The lower limit of this confidence interval that is the curve $\varepsilon_{L1B}^L(t_0)$ ($n = 30$, $V_{\varepsilon_{2B}} = 0.068$, $\alpha = 0.001$, $t = 3.659$ [27], and $\delta_n = 0.0455$ in Figure 10) gives a possibility to determine a safety factor based on the expected value. The accuracy can be improved by assessing the relationship between $V_{\varepsilon_{2B}}$ and t_0 which can be performed in the knowledge of Equation (12).

Figures 11 and 12 show the curves calculated on the basis of those in Figure 10 and the measured creep strain increment values versus the uploading time and the strain load level.

It can be stated that the range bounded by the upper and lower estimations in Figure 10 is also effective in the case of the creep strain to failure increments considering it as a function of the uploading time (Figure 11) or the strain load level (Figure 12).

Figures 11 and 12 confirm the result of analysis that there might exist a maximum for the creep strain to failure increment values when plotting them as a

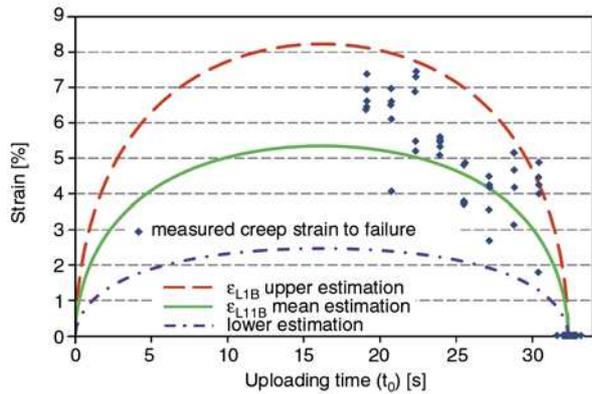


Figure 11. Measured values of the creep strain to failure increments versus the uploading time and their upper, mean, and lower estimations

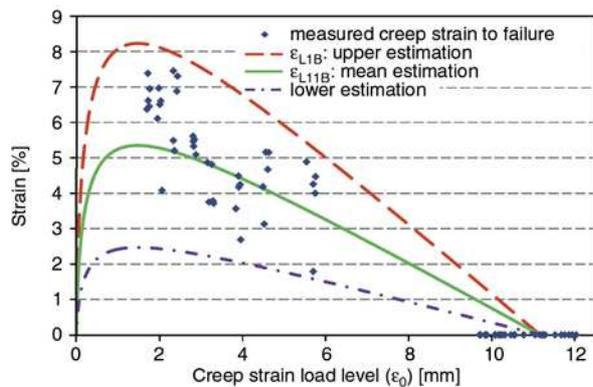


Figure 12. Measured values of the creep strain to failure increments versus the strain load level and their upper, mean, and lower estimations

function of the uploading time or the creep strain load level.

6. Conclusions

In order to study the creep behavior tensile tests and short term creep tests were carried out on injection molded PP specimens at room temperature. After correcting the zero-point error of the tensile measurements the recorded strain-time curves were averaged point- by point. Using an LVE formula developed earlier the LVE creep strain to failure values were estimated from the mean tensile test curve. A Weibull distribution based stochastic model was used to describe the variation of the mean values of these strain to failure estimations where the parameters were found by fitting the measurements. This made it possible to derive an approximating function for the mean tensile test curve as well. By linear variable transformation of the LVE strain characteristics upper, mean, and lower estimations as

well as confidence interval curves for the expected values were determined for the measured creep strain to failure values which give a possibility for designers' calculations at arbitrary creep load levels.

According to the result of analysis the creep strain to failure increment values might have a maximum in the function of the uploading time or the creep strain load level.

The Weibull distribution based mathematical approximation applied gives a hand for developing the non-linear time-transformation for the estimation of the long term creep behavior and the expected lifetime in a next step and a good basis for estimating additional statistical characteristics such as standard deviations and confidence intervals.

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