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ANALYZING THE TENSILE BEHAVIOUR OF FABRICS BASED ON FIBRE BUNDLE MODELS

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Abstract: *Woven fabrics have widely been used for reinforcing polymer composites. Fabrics are built up of yarns or rovings the building elements of which are the fibres. Samples cut out in any main direction of fabrics consist of aligned warp and weft yarns creating yarn bundles and a section of a yarn between two crossing points creates a fibre bundle. The fibre and yarn bundles are kinds of intermediate elements in the fabric as fibrous structure and represent the statistical properties of the structure and its strength. In this paper the so called statistical fibre-bundle-cells (FBC) method was applied to modelling and analysing the tensile behaviour of a fabric made of false twisted multifilament PET yarn and its impregnated form as reinforcement for developing special composite sheets.*

Keywords: *Fibre bundle model, reinforcement, statistical modelling, tensile behaviour*

1. Introduction

Woven fabrics have widely been used for reinforcing polymer composites. Fabrics are built up of yarns or rovings the building elements of which are the fibres. Samples cut out in any main direction of fabrics consist of aligned warp and weft yarns creating yarn bundles and a section of a yarn between two crossing points creates a fibre bundle. The fibre and yarn bundles are kinds of intermediate elements in the fabric as fibrous structure and represent the statistical properties of the structure and its strength such as size effects, damage process, failure and breakage [1-5]. On the basis of that the so called statistical fibre-bundle-cells (FBC) method and software FibreSpace have been developed by the authors [4, 5]. FibreSpace can make the creation of FBC network models easier. In this paper non-linear E-bundles were applied to modelling and analysing the mean tensile behaviour and the failure process of a neat fabric and its coated version. The latter kind of sample made it possible to study the effect of impregnation and coating on the tensile behaviour of yarns and fabrics.

2. Materials and methods

2.1 Materials and tests

The fibrous structures used for testing and modelling were a plain weave fabric made of special false twisted multifilament PET yarns and its coated version (Table 1). The coating was a dicrylan foam of 0.23 g/cm^3 density the recipe of which was: 1000 g Dicrylan PGS, 30 g Knitex CHN, Dicrylan stabilizer FLN, 20 g 25% ammonia. The foam coating was prepared by scraping with a thickness of 300 μm. In order to provide proper basis for modelling and analysis tensile tests were carried out on yarns as well as on neat and coated fabric samples of 50 mm width cut out in directions 0 (warp), 45, and 90 (weft) degrees using gauge lengths of 10 and 100 mm. The instrument was a Zwick Z005 universal tensile tester. In every case the rate of elongation was 100 mm/min and the number of measurements was 5.

On the basis of the measured tensile force and deformation data of yarns and fabric samples tensile strength properties were determined and the mean force-deformation curves were calculated by averaging the measurements point by point.

Table 1. Data of yarn and fabrics

Yarn			**Fabric**				**Coated fabric**	
Material	**Linear density** [dtex]		**Type of weave**	**Yarn density** [1/100 mm]		**Surface density** [g/m^2]	**Coating**	**Surface density** [g/m^2]
	warp	**weft**		**warp**	**weft**			
PET	333		plain	252	240	177	Dicrylan PGS foam	204

2.2 FBC modelling by using non-linear E-bundles and parameter transformations

The FBC modelling method is based on some idealized fibre bundles called fibre-bundle-cells which can be used as building elements of a model network like those in the viscoelastic mechanical models such as spring and dashpot. The weighted parallel connection of them can provide FBC model for describing the mechanical behaviour of a fibrous sample during tensile test. On the basis of an FBC model the deformation and damage processes of fibres and yarns within a fibrous sample can be studied and analysed. Idealized statistical fibre bundles are defined as fibre classes containing the same geometrical (shape, disposition) and mechanical properties (strain state, gripping by the environment). These fibre classes are called fibre bundle cells (FBCs) (Figure 1). In the simplest case the fibres are ideally elastic (E) with linear or non-linear relationship between the strain and load. Both the shape, position, and strength parameters of fibres are assumed to be independent stochastic variables [4-5].

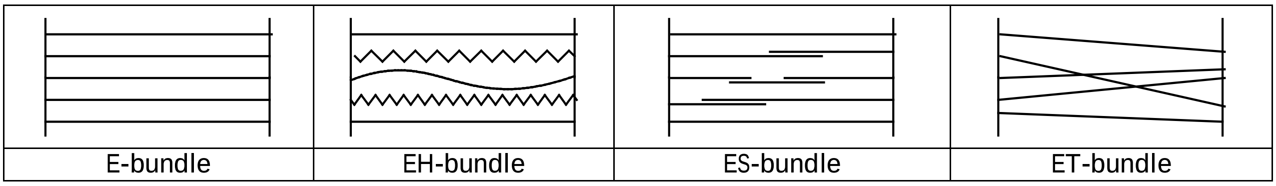
E-bundle
EH-bundle
ES-bundle
ET-bundle

Figure 1. Structural scheme of the idealized fibre bundle cells [4]

The fibres of E-bundle are straight and parallel to the tensile load direction and they are ideally gripped that is they do not slip out of the grips. In the case of the fabric the yarns play the role of the fibre elements. Hence a samples cut out in any main direction of fabrics consist of aligned warp and weft yarns creates yarn bundle. Samples cut out any other direction have more complicated bundle structure.
In this paper for modelling and analyzing the tensile behaviour of samples cut out in main and 45^o directions of fabric a simplified FBC modeling method is used where E-bundles of yarns with nonlinear tensile characteristic can be applied to modeling the fabric specimen (Figure 2).
Cutting out a sample e.g. in warp direction from the fabric is built of warp yarns aligned lengthwise (Figure 2.a) and weft yarns aligned crosswise (Figure 2.b). Loading this sample in lengthwise direction the load is taken up by the warp yarns gripped at both ends and the weft yarns with free ends play just a modifying role by interlacing and crimping the warp yarns. This specimen is considered as an equivalent E-bundle created by fabric yarn elements which describes the mean tensile process of the fabric specimen including the effects of the yarn-yarn adhesion and the possible slippage of the weft yarns (Figure 2.c). Specimens cut out in any direction even if they have more complicated bundle structure can be treated also as a special E-bundle the elements of which are kind of fabric-equivalent yarns.
The expected value of the tensile force process of the E-bundle of yarns (Fig. 1) can be calculated by the following formula [4-5]:

$$\overline{F}_y(u;p_y)=E[F_y(u;p_y)]=f_y(u;p_{y1})\left(1-Q_{\lambda_{By}}\left(u;p_{y2}\right)\right) \tag{1}$$

where u is the bundle elongation, $Q_{\lambda By}$ is the distribution functions of the yarn breaking elongation (λ_{By}) of the yarns, while $f_y(u)$, $f_y(0)=0$, is the normalized tensile characteristic of the yarns, which is linear in simple cases, and parameter vector $p_y=(p_{y1}, p_{y2})$ denote the parameters of $f_y(u)$ and the distribution function of λ_{By}. In Equation (1) the tensile characteristic, $f_y(u)$, describes the failureless work of the fibrous structure while $1\text{-}Q_{\lambda By}(u)$ is a kind of reliability function and represents the statistical properties of the damage process.

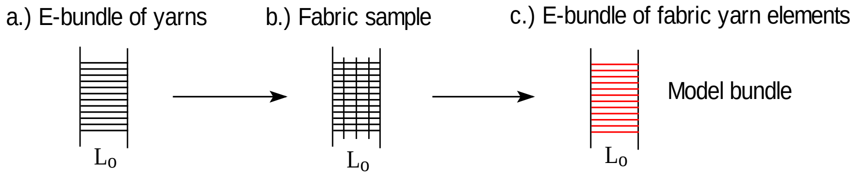

Figure 2. Modelling fabric sample of main direction as an equivalent yarn element bundle

The fabric tested is built up of yarns with the nonlinear characteristics above and the expected tensile process of fabric specimens cut out in a given direction (α) is supposed to be described with a similar formula:

$$\overline{F}_{f,\alpha}(u;p_f(\alpha)) = E[F_{f,\alpha}(u;p_f(\alpha))] = f_y(u;p_{f1}(\alpha))\left(1-Q_{\varepsilon_{By}}\left(u;p_{f2}(\alpha)\right)\right) \qquad (2)$$

where p_f is the new parameter vector, however, the formulae of the tensile characteristic and the distribution function of the fabric specimen are identical with those of the yarn, only the parameters change: $p_y \rightarrow p_f$. According to Equation (2), the fabric specimen in question can be modelled with an equivalent nonlinear E-bundle. Consequently, the properties of the equivalent yarns can be compared with each other in order to understand and analyse by what kind of structural-mechanical and statistical changes the parameter-transformation $p_y \rightarrow p_f$ could be explained. This method discussed above can be used for fabric specimens cut out in any direction. In this case the following rather fexible function was applied to the description of the tensile characterisctic of the specimens of every type:

$$f_y(u;p) = a\left(1-e^{-b(u+u_o)}\right)+c(u+u_o), \quad K = ab+c \qquad (3)$$

where $p=(a,b,c,u_o)$ and a and b are shape parameters, K and c respectively are the initial and asymptotic tensile stiffnesses of the specimen, while u_o is the possible preextension. In addition the breaking elongation yarn elements (λ_B) was assumed to be of normal distribution.

3. Results of measurements

Figures 3 and 4 show the averaged tensile test results for yarn and fabric specimens respectively in the case of 10 mm (left diagram) and 100 mm (right diagram) gauge lengths. The yarn samples were taken from the bobbin, and the neat and coated fabrics in both warp (0 deg.) and weft (90 deg.) directions. Significant difference between the bobbin and fabric yarn can be observed in case of 10 mm gauge length.

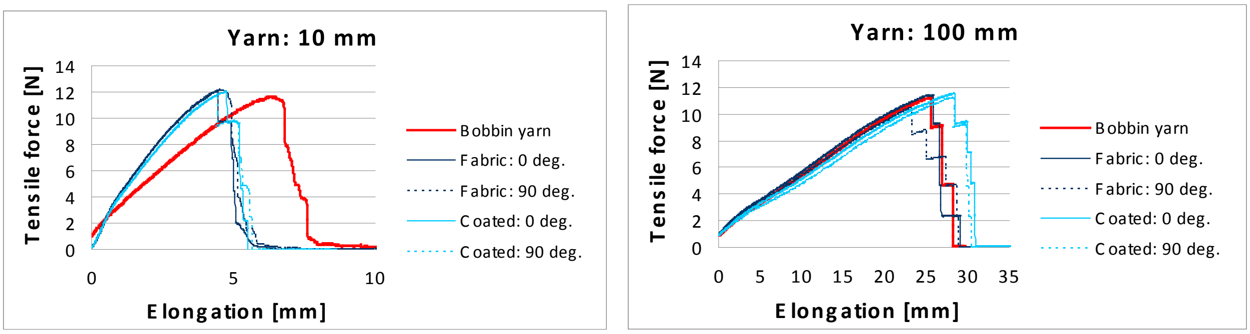

Figure 3. Averaged force-elongation curves measured on yarns taken from bobbin, fabric, and coated fabric

As usual the force-elongation curves in the main and 45 degree directions strongly differ on the other hand the values of both the peak force and the related elongation are larger in warp direction than those in weft direction.

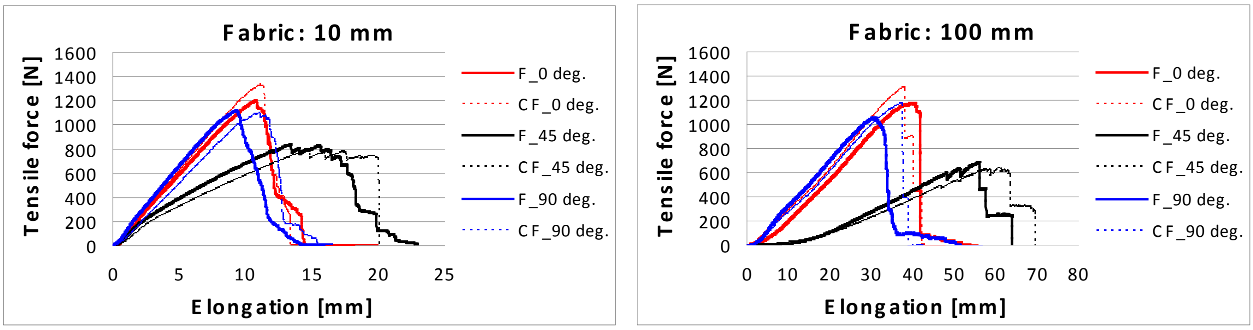

Figure 4. Averaged force-elongation curves measured on neat (F) and coated fabric (CF) in different directions

4. Results of modelling

Using the tensile test results nonlinear E-bundle models were created for modelling the deformation and failure processes of yarn and fabric samples of different gauge lengths (Figures 5 and 6, Table 2).

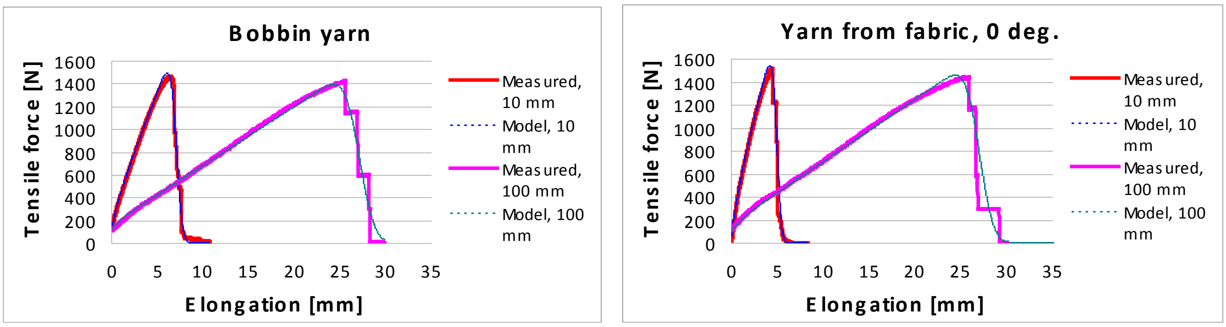

Figure 5. Measured and modelled force-elongation curves of yarns taken from bobbin and fabric in direction 0 degree

Table 2. Results of measuring and modelling yarns

		Yarn from bobbin		Yarns from fabric						Yarns from coated fabric					
	Fabric direction			0 deg.		90 deg.		45 deg. (est.)		0 deg.		90 deg.		45 deg. (est.)	
	Gauge length [mm]	10	100	10	100	10	100	10	100	10	100	10	100	10	100
Model	a [N]	1.70	1.00	2.60	1.00	2.60	1.00	2.60	1.00	2.60	1.00	2.60	1.00	2.60	1.00
	b [1/mm]	1.00	1.50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.50	1.00	1.20	1.00	1.00
	c [N/mm]	1.65	0.41	2.40	0.43	2.30	0.40	2.30	0.41	2.20	0.39	2.20	0.37	2.42	0.38
	Ky [N/mm]	3.35	1.91	5.00	1.43	4.90	1.40	4.90	1.41	4.80	1.89	4.80	1.57	5.02	1.38
	u0 [mm]	0.30	0.60	0.02	0.65	0.02	0.60	0.02	0.60	0.02	0.60	0.02	0.65	0.02	0.60
	Av(λB) [mm]	7.00	27.00	4.88	26.90	5.15	26.80	5.00	26.80	5.15	30.00	5.20	29.90	4.88	29.90
	SD(λB) [mm]	0.50	1.30	0.38	1.20	0.42	1.70	0.36	2.00	0.36	1.45	0.44	0.90	0.38	1.20
	F_peak [N]	1490	1386	1541	1456	1472	1243	1498	1276	1522	1464	1424	1370	1514	1405
	u_peak [mm]	6.06	24.30	4.17	24.37	4.38	23.52	4.32	23.13	4.46	27.00	4.40	27.84	4.18	27.31
	Relative error [%]	3.43	4.07	4.48	4.61	4.85	7.09	3.03	2.07	4.44	4.56	4.98	4.81	4.18	3.68
Measured	AvF_peak [N]	1468	1423	1521	1444	1457	1256	1484	1317	1518	1459	1399	1349	1439	1402
	Avu_peak [mm]	6.27	25.58	4.45	25.85	4.53	23.19	4.41	23.21	4.73	28.37	4.38	28.31	4.39	28.37
Calculated	u_ePO [mm]	4.40		2.03		2.40		2.28		2.05		1.68		1.69	
Corrected	u_peak* [mm]	1.88	21.18	2.42	23.82	2.12	20.79	2.13	20.93	2.68	26.32	2.70	26.64	2.71	26.68
	Av(λ_B)* [mm]	2.60	22.60	2.85	24.87	2.75	24.40	2.72	24.52	3.10	27.95	3.52	28.22	3.19	28.21
	RA(λ_B)* [%]	26.05	22.60	28.54	24.87	27.48	24.40	27.18	24.52	30.98	27.95	35.23	28.22	31.94	28.21
	b* [1/mm]	3.34	1.81	1.84	1.09	2.13	1.12	2.07	1.11	1.77	1.62	1.62	1.28	1.62	1.06
	c* [N/mm]	5.51	0.50	4.41	0.47	4.90	0.45	4.76	0.45	3.89	0.42	3.56	0.39	3.93	0.40
	Ky* [N/mm]	11.19	2.31	9.18	1.55	10.44	1.56	10.14	1.56	8.48	2.04	7.78	1.67	8.15	1.47

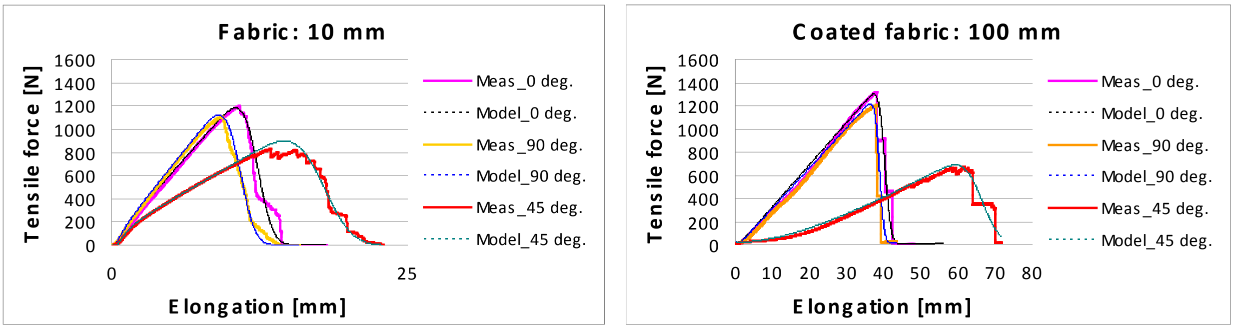

Figure 6. Measured and modelled force-elongation curves of neat and coated fabrics

The averaged yarns curves were calculated for a bundle of so many yarns that corresponded to the fabric specimens. The parameters of the model curves and the force-peak data of the measured averaged curves are summarized in Tables 2 and 3. The model curves fit not only to the increasing part without failure but to that related to the damaging and breaking process as well. The relative mean error of fitting was less than 5% except for the 45 degree directions where it was larger.

Table 3. Results of measuring and modelling neat and coated fabric

		Fabric						Coated fabric					
	Fabric direction	**0 deg.**		**90 deg.**		**45 deg.**		**0 deg.**		**90 deg.**		**45 deg.**	
	Gauge length [mm]	**10**	**100**	**10**	**100**	**10**	**100**	**10**	**100**	**10**	**100**	**10**	**100**
Model	**a [N]**	0.8	-1.1	0.7	-0.45	1	-4	1	-0.18	0.5	-0.185	0.8	-2.6
	b [1/mm]	1	0.26	1	0.7	1	0.045	0.8	1	0.6	1.5	0.6	0.053
	c [N/mm]	0.9	0.29	1.05	0.32	0.48	0.184	0.98	0.289	0.9	0.29	0.4	0.14
	Kf [N/mm]	1.700	0.004	1.750	0.005	1.480	0.004	1.780	0.109	1.200	0.013	0.880	0.002
	u_0 [mm]	-0.5	-0.8	-0.35	-0.6	-0.5	-2	-0.7	-0.6	-0.5	-0.6	-0.5	-1
	Av(λB) [mm]	12.2	41.5	10.7	34.5	17.6	59	12	40	12.3	38.5	19	65.5
	SD(λB) [mm]	1	1.5	1	2	1.9	3.4	0.7	1.3	1.1	1	1.2	3.7
	F_peak [N]	1180	1211	1122	1070	901	684	1318	1290	1082	1204	871	666
	u_peak [mm]	10.47	38.34	9.05	30.73	14.56	53.18	10.65	37.15	10.46	36.19	16.71	58.98
	Relative error [%]	4.39	5.86	2.67	4.62	4.70	6.49	3.74	4.77	4.57	4.23	11.72	6.23
Measured	**F_peak [N]**	1201	1182	1113	1069	837	701	1333	1317	1095	1195	790	654
	Avu_peak [mm]	10.79	40.52	9.30	30.56	13.43	56.41	11.14	38.19	11.03	37.70	14.76	61.31
Calculated	**u_ePO [mm]**	7.02		6.61		8.33		7.42		7.57		9.14	
Corrected	**u_peak* [mm]**	3.77	33.50	2.68	23.94	5.11	48.09	3.72	30.77	3.45	30.13	5.62	52.17
	Av(λ_B)* [mm]	5.18	34.48	4.09	27.89	9.27	50.67	4.58	32.58	4.73	30.93	9.86	56.36
	RA(λ_B)* [%]	51.83	34.48	40.86	27.89	92.74	50.67	45.76	32.58	47.26	30.93	98.56	56.36
	b* [1/mm]	2.86	0.31	3.46	0.89	2.63	0.05	2.40	1.24	1.92	1.88	1.58	0.06
	c* [N/mm]	2.57	0.35	3.64	0.41	1.26	0.22	2.94	0.36	2.87	0.36	1.05	0.16
	Kf* [N/mm]	4.864	0.005	6.062	0.006	3.892	0.005	5.335	0.135	3.833	0.016	2.313	0.003

The models provide the more correct statistical data of the failure of the yarn elements of the E-bundles that are the mean breaking elongation, Av(λ_B), and the standard deviation, SD(λ_B). Besides different comparisons the results of measurements and modelling made it possible to assess the inevitable elastic pulling out of the grips (u_{ePO}) during tensile test which distorted the results and was defined and estimates as follows:

$$u_{ePO} = \frac{10\left(f^{-1}(F_B)+u_o\right)_{10mm} - \left(f^{-1}(F_B)+u_o\right)_{100mm}}{9} \approx \frac{10\left(u_{peak}+u_o\right)_{10mm} - \left(u_{peak}+u_o\right)_{100mm}}{9} \qquad (4)$$

where F_B is a force level close the peak values obtained at gauge lengths 10 and 100 mm, $f^{-1}(F)$ is the inverse function of $f(u)$, u_{peak} is the measured elongation belonging to the force peak. In spite of the wavy notched Vulkolan jaws applied the obtained values of u_{ePO} were rather large, they ranged from 0.85 to 4.57 mm per grip. In the knowledge of u_{ePO} the data could be corrected by using the next formulae (5):*

$$u^*_{peak} = \mathrm{u}_{peak} - \mathrm{u}_{ePO},\; Av(\lambda_B)^* = Av(\lambda_B) - u_{ePO},\; a^* = \mathrm{a},\; b^* = \mathrm{b}\omega,\; c^* = \mathrm{c}\omega,\; b^* = \mathrm{b}\omega,\; \omega = \mathrm{u}_{peak}/\mathrm{u}^*_{peak},\; K^* = \mathrm{K}\omega \qquad (5)$$

Using the definition of u_{ePO} at every force level of the increasing part of the model curves for gauge length 10 and 100 mm the elastic pulling out can be calculated as correction curve (Figure 7) as well.

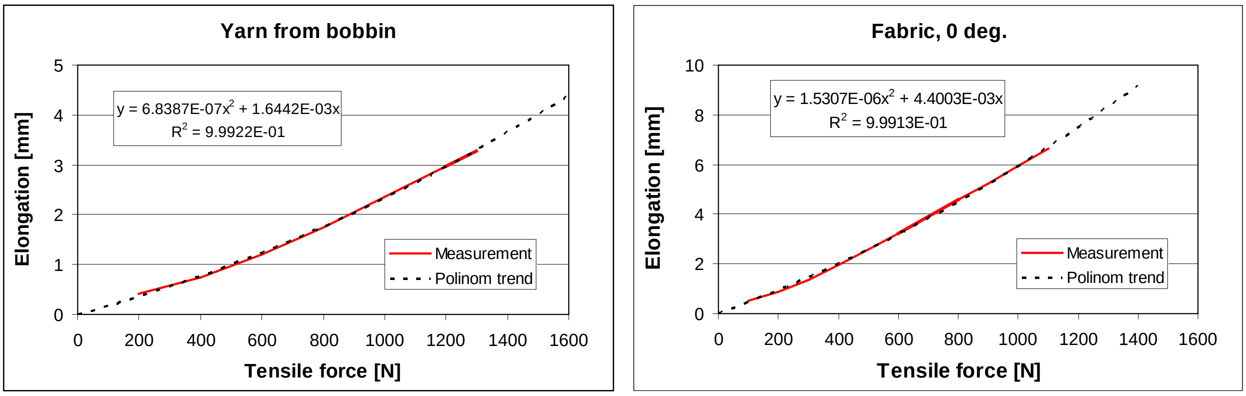

Figure 7. Assessed elastic pulling out of bobbin yarn and fabric specimen during tensile test

5. Conclusion

The FBC models of the yarns and the fabric samples made it possible to study and analyse the deformation and damage processes of them as well as to determine the elastic pulling out of the grips that is inevitable during tensile tests. Among others it could be established that relating to the yarns the tensile stiffness values (both initial and asymptotic) of the fabric specimens are smaller while the breaking elongation of the equivalent yarn elements are significantly larger. In addition the coating did not change significantly the tensile properties of the fabric. Finally, the elastic pulling out of the neat or coated fabric specimens is about two times larger than that of the yarns taken from either bobbin or fabrics.

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