FIBER BUNDLE CELLS BASED MODELLING OF WOVEN REINFORCEMENTS

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Abstract: *Woven fabrics have widely been used in different constructions such as covering sheets or reinforcing polymer composites. Fabrics are built up of yarns or roving the building elements of which are the fibres. Samples cut out in any main direction of fabrics consist of aligned warp and weft yarns creating yarn bundles and a section of a yarn between two crossing points creates a fibre bundle. In this paper the so called statistical fibre-bundle-cells (FBC) method is applied to modelling and analysing the tensile behaviour of a special plain fabric made of false twisted multifilament PET yarn. On the basis of using nonlinear E-bundles made it could be established, that the tensile behaviour of fabric specimen was characterized by that of yarns while the structure of fabric samples cut in warp and 45^o direction had a strong modifying effect.*

Keywords: *Fibre bundle model, reinforcement, statistical modelling, tensile behaviour*

1. Introduction

Woven fabrics have widely been used in different technical constructions such as covering sheets or reinforcing polymer composites. Fabrics are built up of yarns or roving the building elements of which are the fibres. Samples cut out in any main direction of fabrics consist of aligned warp and weft yarns creating yarn bundles and a section of a yarn between two crossing points creates a fibre bundle. The fibre and yarn bundles are kinds of intermediate elements in the fabric as fibrous structure and represent the statistical properties of the structure and its strength such as size effects, damage process, failure and breakage [1-5]. In this paper the so called statistical fibre-bundle-cells (FBC) method developed by the authors [4, 5] was applied to modelling and analysing the tensile behaviour and the failure of a fabric made of false twisted multifilament PET yarn.

2. Modelling method

The FBC modelling method is based on some idealized fibre bundle called fibre-bundle-cells which can be used as building elements of a model network like those in the viscoelastic mechanical models such as spring and dashpot. The weighted parallel connection of them can provide FBC model for describing the mechanical behaviour of a fibrous sample during tensile test, while the serial connection is suitable to model size effects. On the basis of an FBC model the deformation and damage processes of fibres and yarns within a fibrous sample can be studied and analysed. In order to make easier the creation of FBC models a software called FibreSpace has been developed.

2.1. Fiber bundle cells based modeling method

Idealized statistical fiber bundles are defined as fiber classes containing the same geometrical (shape, disposition) and mechanical properties (strain state, gripping by the environment). These fiber classes are called fiber bundle cells (FBCs) (Figure 1).
Fibers of these FBCs are supposed to be perfectly flexible, usually linearly elastic (like in Figure 1) and to break at a random strain (ε_S). They are straight in the E-bundle, loose (ε_o<0) or pre-tensioned (ε_o>0) in the EH-bundle, and oblique (fiber angle $\beta\neq0$) in the ET-bundle, and gripped ideally in these cases. Fibers in the ES-bundle are straight but they may slip out of their grip at a strain level (ε_b<ε_S) or create fiber-chains with

slipping bonds where ε_{bL} is the relative slippage way of fibers. Both the shape, position, and strength parameters of fibers are assumed to be independent stochastic variables [4-5].

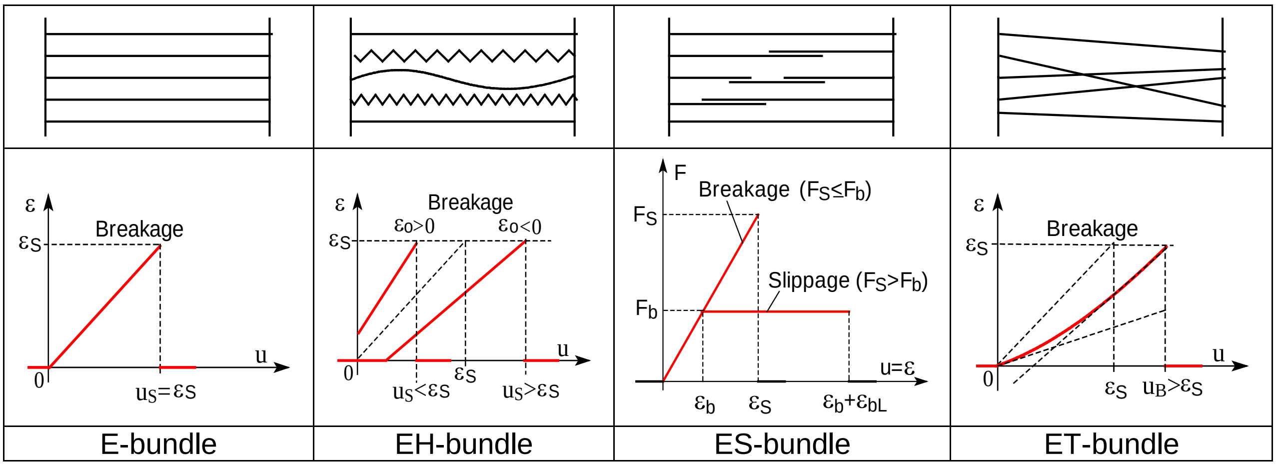


Figure 1. Structural scheme of the idealized fiber bundle cells and relationship between the strains of single fibers and FBCs [4]

In the case of constant rate elongation the tensile force ($F(u)$) creates a stochastic process as a function of the bundle strain (u). Being aware of the relationship between the bundle (u) and fiber strains (ε) (Fig. 1), the expected value of the tensile force of the FBCs ($E(F) = \overline{F}$) could be calculated as a sum of the single fiber forces using the suitable formulas developed. Dividing the expected value by the mean breaking force of fibers, the normalized tensile force of bundle is computed as follows:

$$0 < FH(z) = \overline{F}(z)/n\overline{F}_S \leq 1, \qquad z = u/\overline{\varepsilon}_S \tag{1}$$

where n, $\overline{F}_S$, and $\overline{\varepsilon}_S$ are the number, the mean breaking force and strain of fibers, respectively, and z is the normalized bundle.

FBC modelling provides a suitable bundle or a bundle network with proper parameters which describes the mean tensile process of the given fibre structure with good agreement [4-5].

2.2. FBC modelling of fabrics by parameter transformation

In the case of the fabric the yarns play the role of the fiber elements. Hence a samples cut out in any main direction of fabrics consist of aligned warp and weft yarns creating yarn bundles and a section of a yarn between two crossing points creates a fibre bundles. Samples cut out any other direction have more complicated bundle structure.

In this paper for modeling and analyzing the tensile behavior of samples cut out in main and 45^o directions of fabric a simplified FBC modeling method is used where E-bundles of yarns with nonlinear tensile characteristic can be applied to modeling the fabric specimen (Figure 2).

Cutting out a sample e.g. in warp direction from the fabric built of warp yarns aligned lengthwise (Figure 2.a) and weft yarns aligned crosswise (Figure 2.b). Loading this sample in lengthwise direction the load is taken up by the warp yarns gripped at both ends and the weft yarns with free ends play just a modifying role by interlacing and crimping the warp yarns. This specimen is considered as an E-bundle created by fabric yarn elements which describes the mean tensile process of the fabric specimen including the effects of the yarn-yarn adhesion and the possible slippage of the weft yarns (Figure 2.c). Specimens cut out in a direction different from the main ones though they have more complicated bundle structure can be treated also as a special E-bundle the elements of which are kind of fabric-equivalent yarns.

The expected value of the tensile force process of the E-bundle of yarns (Fig. 1) can be calculated by the following formula [4-5]:

$$\overline{F}_y(u;a) = E[F_y(u;a)] = K_y f_y(u;a_1)\left(1 - Q_{\varepsilon_{Sy}}(u;a_2)\right) \tag{2}$$

where u is the bundle strain, K_y and $Q_{\varepsilon Sy}$ are the initial tensile stiffness and the distribution functions of the yarn breaking elongation (ε_{Sy}) of the yarns, respectively. $f_y(u)$, $f_y(0)=0$, is the normalized tensile characteristic of the yarns, which is in simple cases linear like in Figure 1, while a_1, a_2, and a respectively denote the parameters of the tensile characteristic and the distribution function of ε_{Sy}, as well as the common vector of them. In Eq. (1) the tensile characteristic, $f_y(u)$, describes the failureless work of the fibrous structure while 1-Q(.) is a kind of reliability function and represents the statistical properties of the damage process.

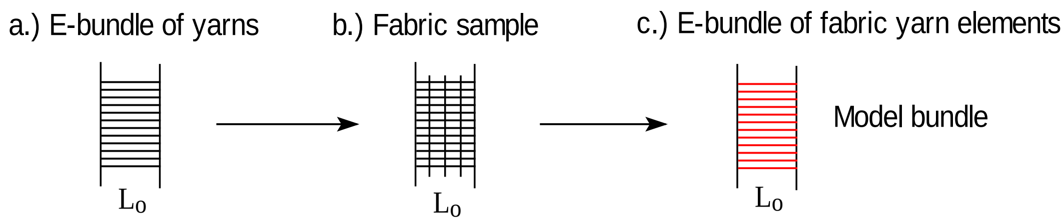


Figure 2. Modelling fabric sample of main direction as an equivalent yarn element bundle

The fabric tested is built up of yarns with the nonlinear characterics above and the expected tensile process of fabric specimens cut out in a given direction (α) is supposed to be described with a similar formula:

$$\overline{F}_{f,\alpha}(u;b) = E[F_y(u;b)] = K_{f,\alpha} f_y(u;b_1)\left(1 - Q_{\varepsilon_{Sy}}(u;b_2)\right) \tag{3}$$

where b is the new parameter vector, K_f is the initial tensile stiffness of the fabric specimen, however, the formulae of the tensile characteristic and the distribution function are identical with those of the yarn, only the parameters change: $a_i \rightarrow b_i$ (i=1,2). According to Eq. (3), the fabric specimen in question can be modelled with an equivalent nonlinear E-bundle. Consequently, the properties of the equivalent yarns can be compared with each other in order to understand and analyse by what kind of structural-mechanical and statistical changes the parameter-transformations $K_y \rightarrow K_f$ and $a \rightarrow b$ could be explained. This method discussed above can be used for fabric specimens cut in any direction.

2.2. FBC modelling of specimen cut in 45° direction

In general, a fabric specimen of width B that is gripped on a tensile tester with gauge length L_o may contain of yarns gripped at both ends (2), or one end (1), or none end (0) (Figure 3). They can be called j-gripped yarns (j=0, 1, 2).

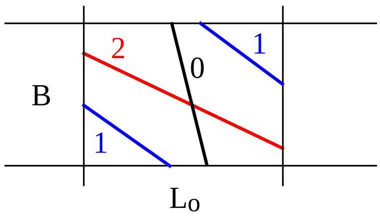


Figure 3. Yarn positions gripped at zero-, one-, and two-ends in a fabric sample

In case of specimens cut in 45° direction the positions of the warp and weft yarns are identical. The bundle structure, however, depends on the gauge length (Figure 4). At small gauge length (L_o<B) there are one- or two-gripped yarns. In case of B<L_o there is no 2-gripped yarn, moreover, if L_o>2B then there is a section in the middle of the specimen where every cross section is intersected by 0-gripped yarns only.

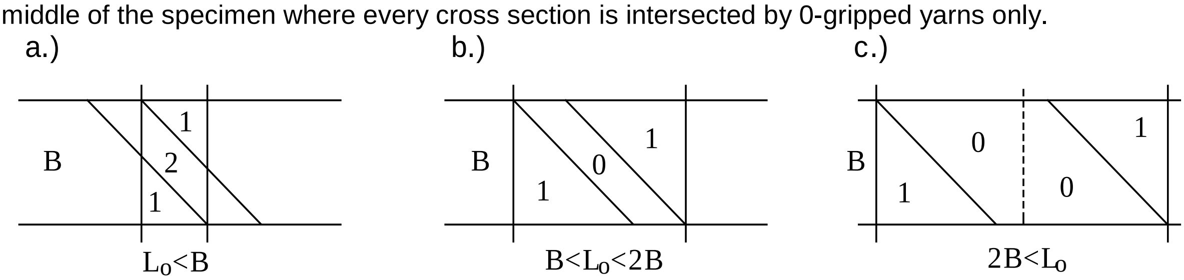


Figure 4. Bundle structure of a fabric sample in 45° direction at different gauge lengths

It is obvious that the weaker cross sections can be found in the middle therefore by analysing these cases it can be shown that increasing the gauge length causes decrease in the tensile strength of the specimen. This is a structural size effect on the strength. There is a joint size effect as well, caused by the stochastic nature

of the structural elements and the imperfection of the fabric structure. Using FBC technique both types of size effects can be modelled.
In this paper the aim of FBC model based analysis is to study the applicability of the parameter transformation of FBC models described above in the case of small gauge lengths. Hence let us examine the bundle structure and the mean tensile strength for case a.) in Figure 4.
The failure of a yarn can take place in two ways: by breakage or slippage out of the gripping provided by the crossing yarns. For simplicity this latter effect of adhesion type can be given as proportional to the slipping length (*l*) like using the so called Kelly-Tyson relationship in composites [6]. In accordance with that the resistant force (F) against slipping is as follows:

$$F = fl \tag{4}$$

where f [N/mm] is the specific resistance. If this gripped yarn length is less than the so called critical adhesion length ($l<L_S$) the yarn slips out of the gripping, however, it is equal to or greater than L_S, then the yarn breaks. In short fibre reinforced composites $L_C=2L_S$ is called critical fibre length [6]. If the adhesion resistance as specific force is not constant along the yarn section in question then its integral gives the resistance force. Hence, for example, if breakages of 1-gripped yarns occur that is $L_S < L_o\sqrt{2}$ then a part of the 1-gripped yarns and every 2-gripped yarn break (Figure 5).

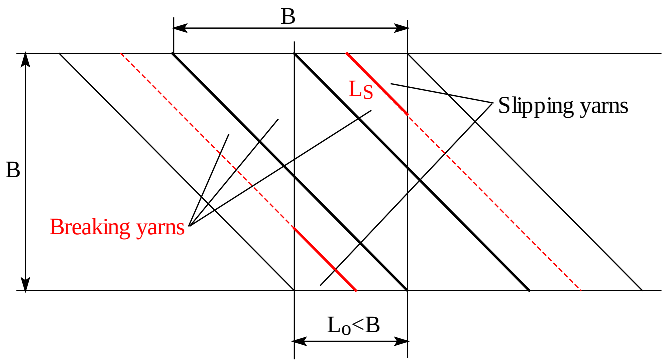


Figure 5. Bundle structure of a fabric sample in 45° direction if the gauge length is small and the adhesion between yarns is rather large

In the knowledge of the mean breaking force of the yarns (F_S) obtained by tensile tests on single yarn samples the critical adhesion length can be determined and understood as follows:

$$F_S = fL_S = f_G L_{GS} \tag{5}$$

where $f_G=f+f_o$ is the specific resistance valid for yarns within the grips and $f_o>0$ is the increase related to that in the free parts of the sample arising from the pressing effects in the grips. Consequently, the critical adhesion length (L_{GS}) in the grips is significantly larger than L_S and so is f_G related to f.
Considering e.g. the warp yarns, the free part of the gripped fabric sample (Figure 5) can be composed of two types of yarn bundles: the 2-gripped warp yarns create an oblique E-bundle, that is a special ET-bundle, while the 1-gripped yarns can be modelled an oblique ES-bundle, that is a special EST-bundle, in which the yarns longer than L_S break and those smaller than L_S slip. Because of the symmetry the same is true for the weft yarns. The parallel connection of these bundles gives an FBC model which of course provides a more precise description than that when using the simple nonlinear E-bundle model. It can be noted that the former FBC model is a kind of decomposition of the latter.

3. Materials and measuring methods

The fabric used for testing and modelling was of plain weave and made of special false twisted multifilament PET yarn (Table 1). In order to provide data for modelling tensile tests were carried out on warp and weft yarn samples taken out of the fabric and fabric samples cut out in the machine and 45° directions. The instrument was a Zwick Z005 universal tensile tester that provided the measured data in electronic format. The gauge length was 10 mm both for yarn and fabric samples. Using the same rubber coated jaw faces and grips and constant rate extension mode in every case the test speed was set at 100 mm/min.

Yarn			Fabric		
Material	**Linear density** [dtex]		**Type of weave**	**Yarn density** [1/100 mm]	
PET	warp	weft	plain	warp	weft
	333			252	240

Table 1. Data of fabric examined

4. Results of measurements

4.1. Tensile test of yarns

In Figure 5 results of tensile tests obtained for warp and weft yarns can be seen, where the yarns was taken out of the fabric. Averaging them point by point provided a load-elongation curve which is equivalent with that of an E-bundle of the same yarn.

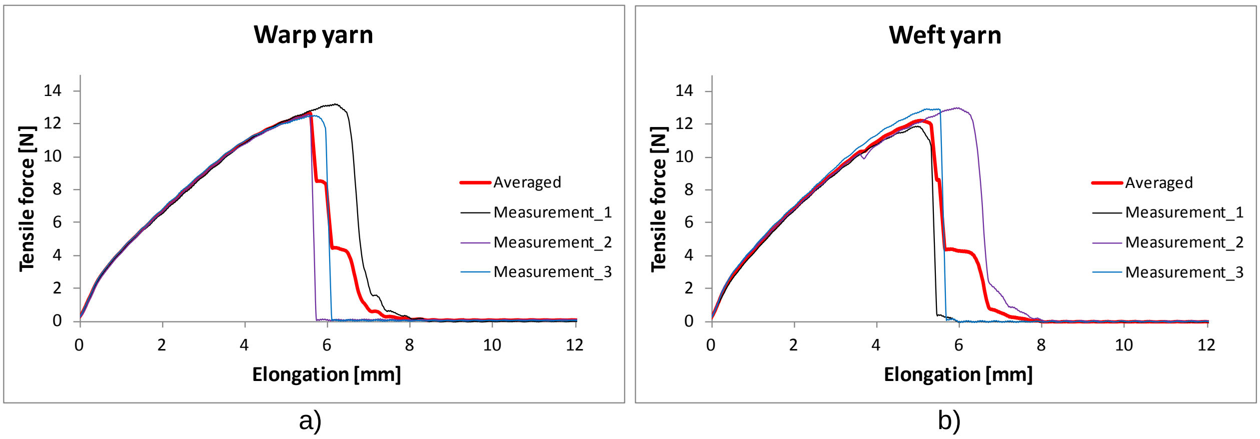


Figure 5. Tensile test measurements performed at gauge lengths 10 mm on warp (a) and weft (b) yarn samples

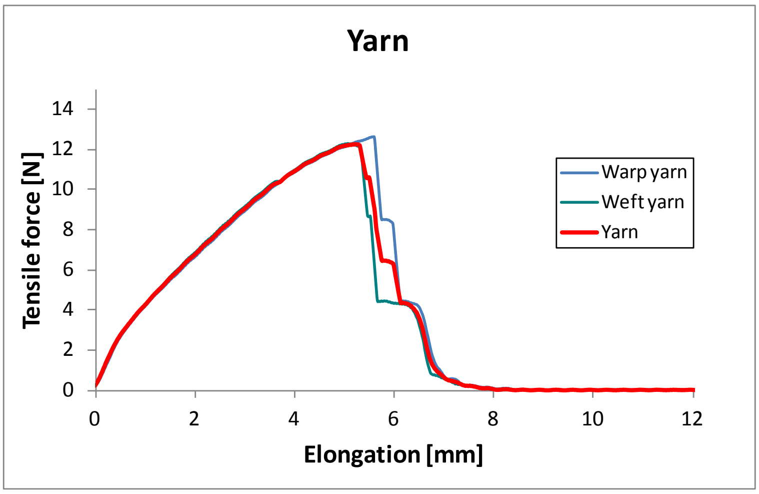


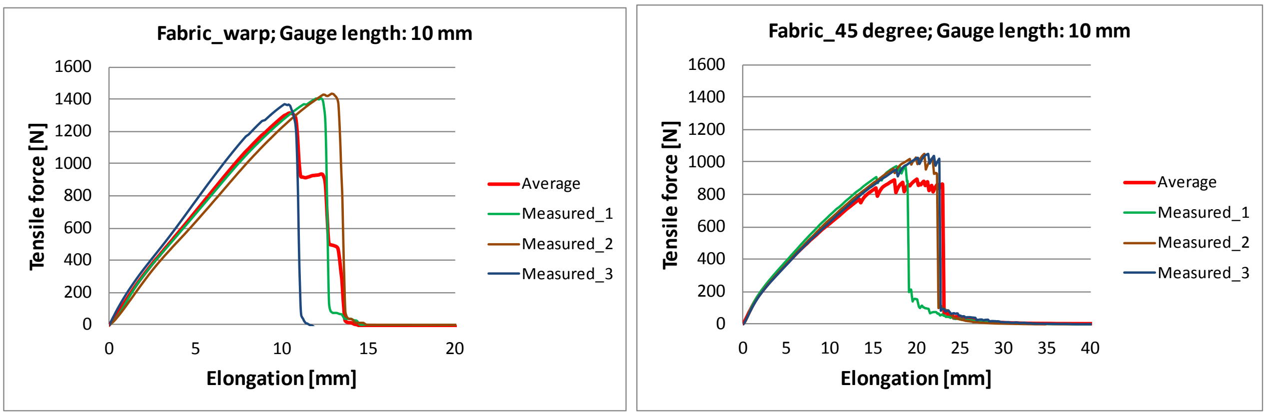
Figure 6. Averaged tensile load versus elongation curves of the warp and weft yarns and their average

Figure 6 shows the two mean curves and their average. Since considering the onset parts the deviation between the mean curves of the warp and weft yarns is negligible and the failure processes are similar therefore their average was used for further analysis.

4.1. Tensile test of fabric specimens

In Figure 7 the tensile load – elongation relationships measured on fabric specimens cut in warp and 45^{o} directions as well as their average are plotted.

As it is visible in Figure 7 the force values measured are remarkable smaller in 45° direction. Multiplying the average curve of yarns by the number of yarn in the warp directed fabric sample (Table 1) resulted in a force-elongation curve equivalent an E-bundle of yarns. These curves can be seen together in the same diagram in Figure 8 representing that the yarn bundle consisting of independent and parallel aligned elements shows larger tensile stiffness and strength, at the same time smaller deformability than the fabric samples.



a) b)

Figure 7. Tensile test measurements performed at gauge lengths 10 mm on fabric samples cut in warp (a) and 45° (b) directions

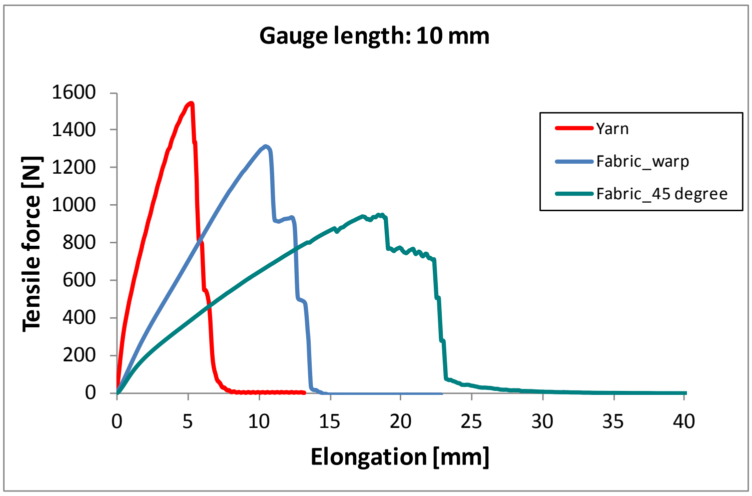


Figure 8. Tensile test measurements performed at gauge lengths 10 mm on yarn and fabric samples, the latter in warp and 45° directions

5. Evaluation and discussion of results

5.1. FBC model of yarns

As mentioned in Chapter on the basis of the averaged tensile force-elongation curve of yarns an E-bundle model created of yarns with nonlinear tensile characteristic as follows:

$$k(u) = cu + a\left(1 - e^{-bu}\right) = K_y\left(\frac{u}{K_y} + \frac{a}{K_y}\left(1 - e^{-bu}\right)\right) \tag{6}$$

where K_y is the tensile stiffness:

$$K_y = c + ab \tag{7}$$

For modelling the failure process normal distribution was used:

$$Q_{\varepsilon_{Sy}}(u) = \Phi\left(\frac{u - m_y}{\sigma_y}\right) \tag{8}$$

where m_y and σ_y are the expected value and the standard deviation of the yarn breaking elongation.
In Figure 9 the tensile characteristic (TCh) and the model curve can be seen showing that the latter is in good agreement with the measured average curve. From fitting the model parameters are K_y=2.1 N/mm, a=2.75 N/mm, b=1.6/mm, m_y=5.9 mm, σ_y=0.65 mm, hence the variance of the latter is 11%.

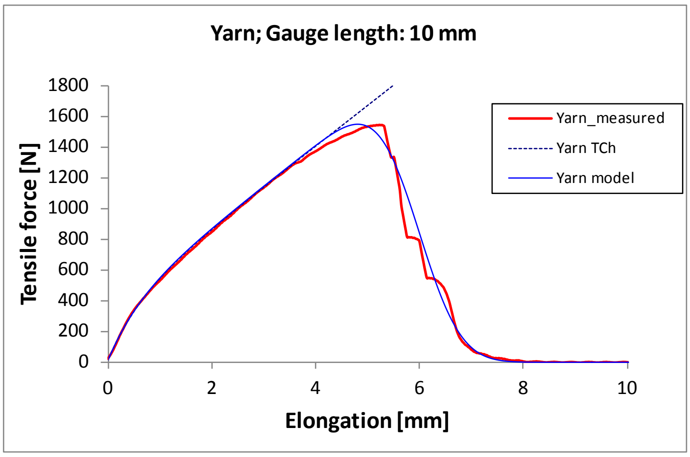
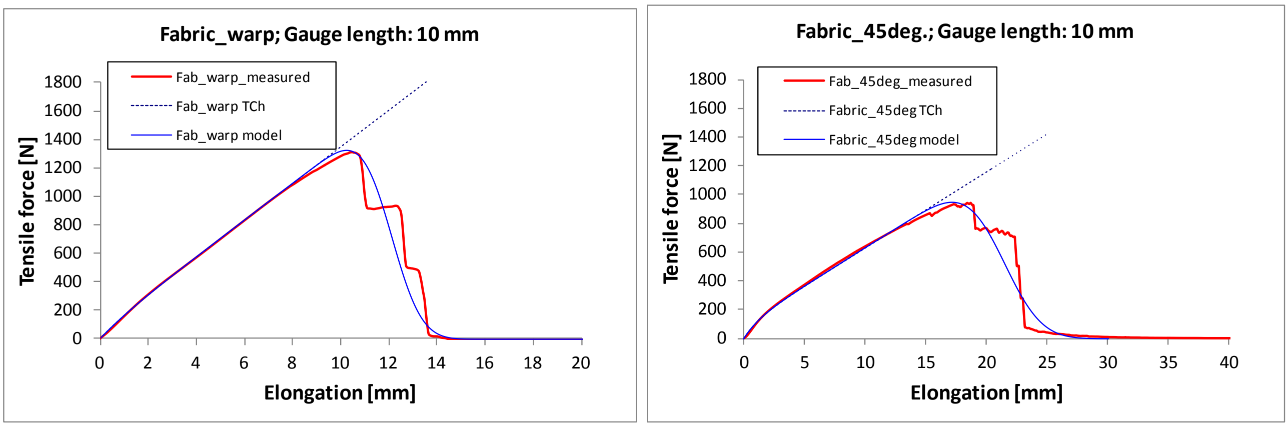


Figure 9. The averaged force-elongation curve of 3 measurements performed at gauge lengths 10 mm on yarn samples and the FBC-model-curve

5.2. FBC models of fabric specimens

Figure 10 shows the graphical results of FBC modelling the tensile force-elongation measurements of fabric samples cut in warp and 45^o direction. Table 2 contains the model parameters obtained by fitting the expected tensile process of the nonlinear E-bundle according to Eqs. (3), (6), and (7).



a) b)

Figure 10. The averaged force-elongation curves of 3 measurements performed at gauge lengths 10 mm on on fabric samples in warp and 45^o directions and the FBC-model-curves

By the results the basic assumption for the modelling method was proven because the same type of E bundle model including the tensile characteristic and the failure process could be applied to both the yarn and the fabric specimen cut in different directions. All that means that basically the yarn determines the tensile behaviour of the fabric samples tested
The initial tensile stiffness of the yarns, K_y, is closely 5 times larger than that of the fabric in warp direction while this ratio concerning the steepness of the next essentially linear onset part (c) is only about 2 times (Table 2).

	Yarn	Fabric	
		Warp	45 degree
a [N]	2.75	0.5	0.8
c [N/mm]	2.1	1.02	0.42
b [1/mm]	1.6	0.8	0.8
K[N/mm]	6.5	1.42	1.06
m [mm]	5.9	12	21
σ [mm]	0.65	1	2.5
CV [%]	11	8	12

Table 2. Parameters of the FBC models

This refers to that the 2-gripped yarn elements having some crimping in the fabric are not strained while after the straightening at proper load the smaller difference can be explained by structural deformation of the fabric. Comparing the warp and 45^o directions of the fabric, there are significantly smaller differences in the stiffness values of the onset parts. The initial stiffness in warp direction is larger by about 40% while that of the next onset part is larger more than 2 times as a result of the shearing effects occurring on the 1-gripped yarns in the first line. The constants, *a* and *b*, determine the shape of the initial curved part of the force-deformation relationship and they are close to each other in the two fabric directions.
The shape of the reliability function and the coefficient of variation (CV in Table 2) is similar in every case, that is the damage process basically takes place in a similar way meaning that the majority of the failures are breakages of yarn elements, however, especially in the 45^o direction the considerable increase of the standard deviation (σ) can be explained by the slippages of the 1-gripped yarns (Figure 5) caused by shearing effects. Considering the deformation of yarns and the fabric in warp direction the large difference in the mean breaking elongation values (m) can be referred to the structural differences moreover the elastic pulling out of the grips should be taken into account while the also large difference between the warp and 45^o directions is caused by the different bundle structure of tha samples and the shearing effect caused in the case of the 1-gripped yarns in the latter case.

6. Conclusion

A nonlinear E-bundle based evaluation method of the tensile strip test of fabric was developed for modeling the mean force-elongation curve at the same gauge length and analyzing the structural-mechanical behavior of specimens tested.
As an application samples of 50 mm width and 10 mm gauge length cut out of plain fabric made of false twisted multifilament PET yarns were tested on tensile tester in warp and 45^o directions. The results of measurements were compared to those obtained on yarn samples of the same gauge length. The comparison performed by using the E-bundle based modelling method proved that the FBC models made it possible to study and analyse the deformation and damage processes of the fabric samples. On the basis of using nonlinear E-bundles made it could be established, that the tensile behaviour of fabric specimen was characterized by that of yarns while the structure of fabric samples cut in warp and 45^o direction had a strong modifying effect.
It is planned that in the future this model providing a global description on the deformation and failure process will be decomposed by using the parallel connection of ET- and EST-bundles in order to obtain a more detailed description.

7. Acknowledgements

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