



## STRUCTURE AND STRENGTH ANALYSIS AND MODELLING OF YARNS OF FABRIC REINFORCEMENT OF FLEXIBLE COMPOSITE SHEET

Alexandra Csilla KOVÁCS<sup>1</sup>, Zsuzsanna Anett HUSZÁR<sup>1</sup>, Bidour AL-GAADI<sup>1</sup>,  
Marianna HALÁSZ<sup>1</sup>, László Mihály VAS<sup>1</sup>, Péter TAMAS<sup>2</sup>, Omar CHERKAOU<sup>3</sup>,  
Mohamed DALAL<sup>3</sup>

Budapest University of Technology and Economics, Faculty of Mechanical Engineering

<sup>1</sup>Department of Polymer Engineering

<sup>2</sup>Department of Mechatronics, Optics and Engineering Informatics

The Advanced School of Textile and Clothing Industry of Casablanca

<sup>3</sup>Research and Development Laboratory

### **Abstract:**

*The composites play a very important role in engineering practice. The flexible, sheet-like composite can be found in the building industry, both in structural elements of the roof and as the awning-cloth of trucks, various protective tarpaulins or tents etc. To model the flexible, fabric reinforcement of the composite sheets, the elements of the structure should be analyzed, namely the yarn, the fabric and the composite itself. In this paper the main goal was to test the geometry and mechanical properties of the yarn to create a base for a later modelling of the whole coated woven fabric. The tested material was a textured polyester multifilament yarn. The fibres and the structure of the yarn were analyzed by electron microscope, and tensile tests were performed at different gauge lengths of yarn in order to study how the Peirce model can be fitted to our yarn. To get the appropriate fit, a so called "Peirce zone" was created based on the Peirce model.*

### **Keywords:**

*flexible composite sheet, PES multifilament yarn, tensile test, Peirce model, material modelling, material parameter*

## **1 INTRODUCTION**

The flexible sheet-like composites are made of PVC in that way that the polyester reinforcement is covered with PVC on both sides and PVC penetrates among the fibres [1-3]. Canvas structures are often used for architectural and engineering establishments. Canvas products are characterized by considerable mobility and they need little maintenance. Besides they have good strength characteristics, they are also cheap and light, and they do not use much space if they are folded. They are suitable for the building of huge span structures such as enormous halls and stadiums (Figure 1). During past decades the materials applied for canvas structures developed a lot. Still, one of the most widespread materials is also the polyester, or perhaps the polyamide fibre reinforced PVC.



*Figure 1 Olympic Hall in Munich (built in 1972) and a Stadium in Florida in Rays Ballpark, which has been building in the classical structure of Munich Olympic Hall (2012) [1-2]*

Nowadays the designing of the products takes place with the help of computerized designing systems in almost every area. This needs a model which describes the mechanical behaviour of the applied material. When setting up the material model of flexible sheet-like composite it is necessary to test and model every level of the structure. Therefore, what should be examined are the elementary fibres and yarn, the fabric made from these, and also the canvas made by covering the material with PVC.

The long-range aim of the research is to model the geometry and mechanical behaviour of the canvas structure that is based on the complete examination of the building elements. As the result of the establishment of the Hungarian-Moroccan TÉT research programme every structural element of the canvas can be examined because the partner Moroccan university has the suitable material and technology for producing both the fabric and the composite sheet.

The aim of the present study is the first step of this complex research, namely, the structural and strength examination of the building elements of the flexible composite sheet such as the polyester fibres and yarn.

## **2 STRUCTURAL EXAMINATIONS**

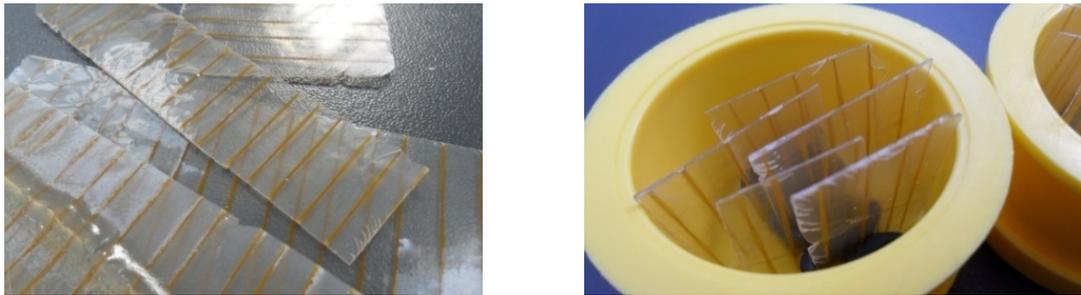
The aim of structural examinations is to determine the typical characteristics of the textured polyester multifilament yarn. The structural examinations were carried out with the help of optical and electron microscopy.

### **2.1 Used equipment**

An Olympus BX 51M optical microscope provided with a computer aided CCD (Charge-coupled device) camera system and the AnalySIS image processing programme and a JEOL JSM 6380LA scanning electron microscope (SEM) were applied to the structural examinations. To cover the surface of the disc test bodies in order to make the surface conductive a JEOL JFC-1200 type gilder equipment which was used.

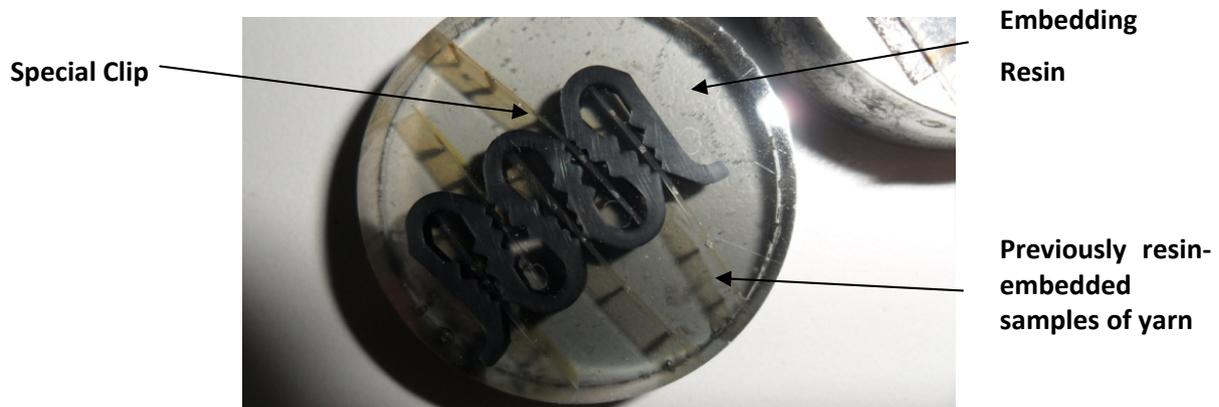
### **2.2 Preparation of sample**

For making the electron microscopic recordings, a ready-made special test body was needed. The preparation of the test body which contained the examined yarn happened in more phases. As a first step, yarn sections selected for the examination were laid parallel to each other on a transparent plastic sheet (Figure 2), then a coat of thermosetting resin was put on them, which cross-linked in one day. Following that, this resin sheet containing the yarns samples was separated from the transparent plastic sheet, then little squares were cut from it. The little square-like plates were put in a cylinder shape with the help of a special snake-like tweezers.



*Figure 2 Preparation of sample: yarns embedded into resin*

The cylinder was filled up with thermosetting resin. By the next day the resin cross-linked and the sample was finished with the help of a polishing machine in order to make the suitable surface (Figure 3).



*Figure 3 The finished resin disc specimens*

The last step of preparing the discs for electron microscopic examination was the gilding. In the course of gilding some nanometres thin gold coat was put on the surface of the sample with the help of a cathode pulverizer. This thin gold coat makes the surface of the sample electrically conductor, which is necessary for the examination, but does not deform the surface structure of the sample.

### **2.3 Results of the examination**

The geometrical examination of the yarn and the elementary fibres were done both on electron microscope and optical microscope. The average number of fibres in the yarn, the shape and size of the cross-section of the yarns, the diameters of the yarns, and other structural characteristics of the yarn structure were determined. The number of fibres in the yarn cross section calculated from the linear density was compared with those counted under the microscope. It can be well seen in the SEM micrograph that the cross section of the elementary fibres is a polygon and in most cases an irregular hexagon (Figure 4). The average fibre number was 92 on the basis of the examined cross-sections of the yarn. The mean linear density of the yarn was 37.5 tex, while that of the elementary fibres was 0.41 tex. According to the examinations two sections with different structures changed each other along the yarn with a period of 12 mm: they were approximately 4 mm long false twisted and 8 mm long untwisted parts (Figures 5 and 6). The average diameter of the twisted parts was 334 micrometres, and that of the untwisted parts with loose fibres came about 575 micrometres, one and a half greater than the previous value.

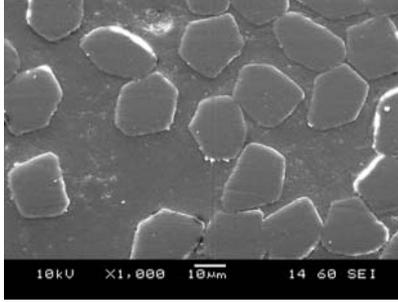


Figure 4 Cross section of filaments (SEM)

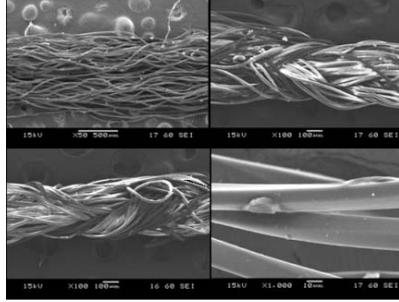


Figure 5 The structure of yarn (SEM)



Figure 6 The structure of yarn in optical microscope

### 3 STRENGTH EXAMINATIONS

After the geometrical examination of the yarn tensile tests were carried out at different gauge lengths and it was investigated how to apply the Peirce's theory [4-6] to describing the size effect on the yarn strength.

#### 3.1 Theoretical basis – Peirce's model of the length depending yarn strength

As it is known, the mean breaking force of the yarns decreases with increasing the gauge length. Based on the "weakest link" principle and measurements done at a small gauge length named basis length ( $l_0$ ), the Peirce-model of yarn strength provides the estimation of the mean strength ( $\bar{F}_s$ ) and standard deviation ( $S_{F_s}$ ) for gauge lengths  $nl_0$  ( $n \geq 1$ , integer). Supposing that the yarn can be considered as a chain built up of independent links of length  $l_0$  and using some other special assumptions and neglects Peirce summarized his theory in the following equations [4-6]:

$$\bar{F}_s(nl_0) = \bar{F}_s(l_0) \left[ 1 - 4,2 \cdot V_{F_s} + 4,2 \cdot V_{F_s} \cdot \left(\frac{l_0}{nl_0}\right)^{1/5} \right] \quad (1)$$

$$S_{F_s}(nl_0) = S_{F_s} n^{-1/5} \quad (2)$$

From Equation (1) the next relations follow:

$$F_s(nl_0) \xrightarrow{n \rightarrow \infty} F_s(\infty) = \bar{F}_s(l_0) (1 - 4,2 \cdot V_{F_s})$$

$$\bar{F}_s(nl_0) \xrightarrow{n \rightarrow 0} \infty$$

where  $V_{F_s} = S_{F_s} / \bar{F}_s$  is the relative standard deviation.

However, it could be seen from some examinations that the formula above significantly underestimated the average breaking force belonging to longer sections [5]. If the value of  $n$  is defined as  $n=l/l_0$ , then equations (1) and (2) can be expanded also to the positive real numbers. Introducing constants  $a$ ,  $b$ ,  $\gamma > 0$ , and  $0 < c < 1$  then equations (1) and (2) can be modified in the following way [5]:

$$\bar{F}_s(nl_0) = \bar{F}_s(l_0) \left\{ 1 - a + b[(c+1)/(c+n)]^\gamma \right\} \quad (3)$$

$$S_{F_s}(nl_0) = S_{F_s}(l_0) [(c+1)/(c+n)]^\gamma \quad (4)$$

Equations (3) and (4) are called the modified Peirce-model the domain of which is  $0 \leq n=l/l_0 < \infty$ .

Introducing a new variable,  $X_n$ , and constant  $\gamma$  is chosen as 1/5 as it is in Peirce's equations, then Equation (3) becomes linear and the coefficients can be determined as linear regression parameters [5]:

$$\underbrace{\overline{F_S}(nl_0)}_Y = \underbrace{\overline{F_S}(l_0)(1 - aV_{F_S})}_A + \underbrace{\overline{F_S}(l_0)bV_{F_S}}_B \cdot X_n \quad X_n = [(c+1)/(c+n)]^\gamma \quad (5)$$

It can be noted if  $a=b=4.2$  and  $c=0$ , then Equation (5) gives the original Peirce-case.

### 3.2 Measuring results

The tensile tests were carried out on a Zwick Z005 type universal tensile tester controlled by a computer. Its technical parameters are: the crosshead speed of is 0.0005-3000 mm/min, the maximum load capacities of the machine and the applied measuring cell are respectively 5 kN and 20 N, and finally, its sensitivity is 0.00001 N.

The examinations were performed with the crosshead speed of 100 mm/min, because at this speed the deformation processes were measurable, but the measurement was not too time-consuming. At the gripping of the yarn samples a preload of 0.1 N was applied. Table 1 contains the number of measurements that were done on the yarns of different length and Figure 7 presents the average, the minimum, and the maximum breaking forces as a function of gauge lengths.

Table 1 Number of measurements

Gauge length [mm]	1	10	50	75	100	200	300	500
Number of measurements [-]	20	100	50	20	30	20	20	20

In Figure 7 it can be seen that the breaking forces belonging to the gauge lengths of 75 mm, 200 mm, and 500 mm do not follow the expected trends. It is known that a yarn of a given length always breaks at its weakest cross section therefore the mean breaking force usually decreases as a function of gauge length. Namely, if the given yarn section is a part of a longer section, the breaking of the yarn either happens at the weakest point of the given yarn section or at an even weaker place of the longer section, to be more specific, at the smaller breaking force.

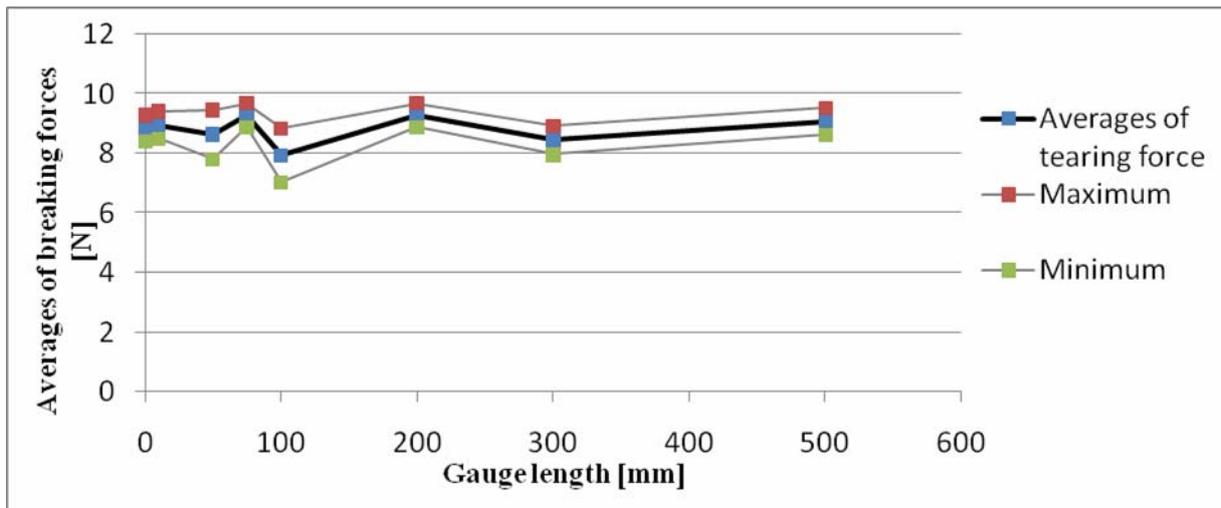


Figure 7 Averages of breaking force depending on gauge length

### 3.3 Evaluation of results using the original and modified Peirce-models

During the evaluation of the results, first it was examined that to what extent the measured data fit to the original and the modified Peirce-model.

For this reason  $l_0=10$  mm was chosen as base gauge length and the most measurements were done at that as a basis for the original Peirce model. The measurements at 1 mm gauge length were performed only for comparison. The modified Peirce-model was determined from the data belonging to gauge lengths of 10, 50, and 100 mm, where the  $X_n$  values were calculated at  $c=0.1$ . From determining the linear trend  $Y=A+B \cdot X_n$  the regression parameters came about  $A=6.531$  and  $B=2.488$ . Table 2 contains the measured and calculated values.

Table 2 Peirce model values

Gauge length [mm]	0	1	10	50	100	500
<b>n</b>	0	0.1	1	5	10	50
<b>Mean measured breaking force, <math>F_s</math> [N]</b>		8.83	8.95	8.63	7.93	9.07
<b><math>X_n</math> [-] (<math>c=0.1</math>)</b>	1.62	1.41	1.00	0.74	0.64	0.47
<b><math>F_{s\_modified}</math> Peirce [N] (<math>A=6.531</math>; <math>B=2.488</math>)</b>	10.55	10.03	9.02	8.36	8.13	7.69
<b><math>F_{s\_original}</math> Peirce [N] (<math>a=b=4.2</math>; <math>c=0</math>)</b>		11.21	8.95	7.88	7.52	6.85

Figure 8 shows the measured points and the curves calculated using the Peirce-model and its modification.

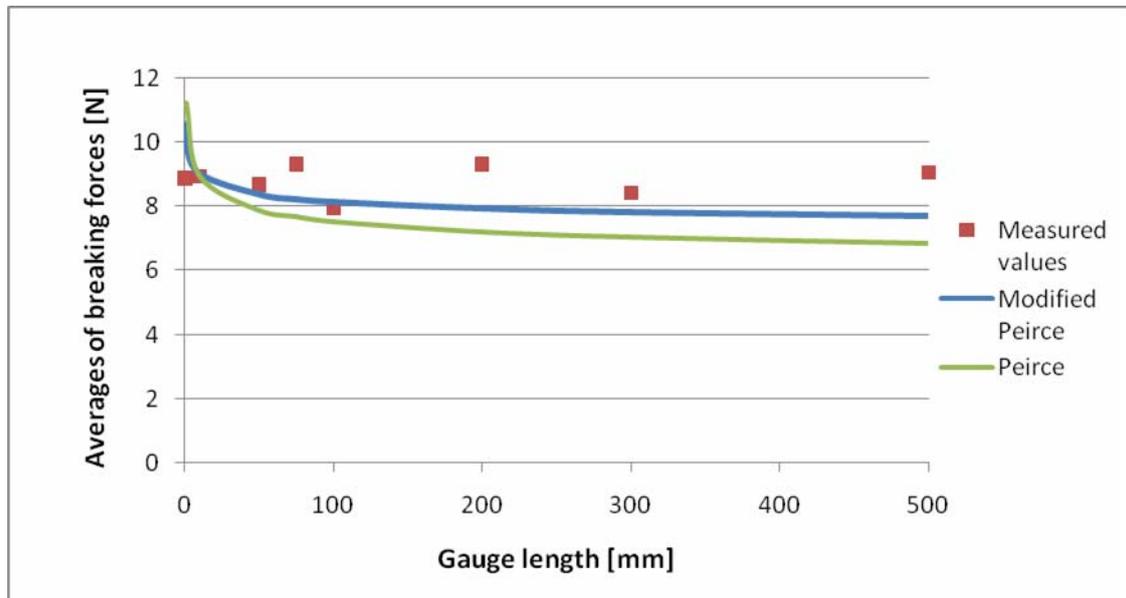


Figure 8 Measured points and the original and modified Peirce-curves

It can be seen that the measured points do not fit well neither to the original nor to the modified Peirce-model (Figure 8). What is more, it also cannot be said whether they follow any kind of tendency, they seem to be random. Taking into consideration the reasons of the deviations the followings appeared:

- Peirce built up his theory on yarns homogeneous along their length (containing identical elements) [4-5]. The presently examined yarn is not homogenous because twisted and untwisted parts follow each other.
- The mistakes arising from the gripping should not be left out of consideration. It happened several times that the yarn broke at one of the gripping jaws.
- During the gripping of the yarns twisted or untwisted yarn sections randomly got into the grips.

The Peirce model could be modified further on so that it fitted better the measured results based on analysing the relation between the inhomogeneous parts of the yarn and the gripping mode.

### 3.4 The separation of the Peirce-model and its application to the examined yarns

During the analysis of the measuring data, it arose that if not to a curve, but to a zone the measured, strongly unstable yarn strength values could fit. For this reason, it was tried to find lower and upper values of the measured data and to fit a *Peirce zone* bounded by an upper and a bottom curve.

As a first step, the values of the breaking forces  $F_S(l_0)$  were separated into two sets: upper and bottom data denoted by  $F_{S,Upp}(l_0)$  and  $F_{S,Bott}(l_0)$  respectively were the values above and below the mean. Then their mean values and standard deviations were determined, and finally, the upper (*Peirce-Upp*) and bottom (*Peirce-Bott*) Peirce-estimations were calculated. Figure 9 shows clearly that in comparison with the earlier modified Peirce-curve (yellow dashed line), the Peirce-zone, which is bounded by upper and bottom Peirce-curves (dark and light blue dashed line), is closer to the measured points, but it does not include all the measured points.

In order to get a better fitting modified Peirce-zone the convex linear combination of the upper and bottom mean breaking force at the  $l_0$  gauge length ( $\overline{F_S}(l_0)$ ) and the bottom and upper Peirce-curves ( $\overline{F_{S,Peirce}}(n \cdot l_0)$ ) respectively were calculated according to Equation (6) below [5, 6].

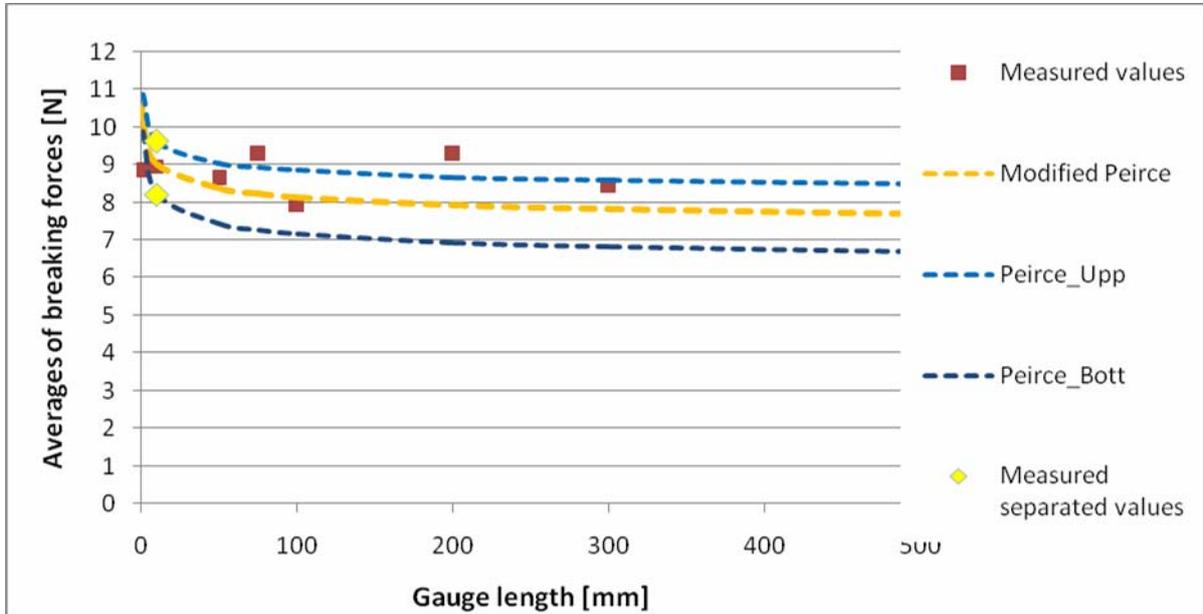


Figure 9 The Peirce-zone

$$\hat{F}(nl_0) = \alpha \overline{F_S}(l_0) + (1 - \alpha) \overline{F_{S,Peirce}}(nl_0) \quad (6)$$

where the  $\alpha$  and  $(1-\alpha)$  are kind of dependence and independence factor, respectively. Thus, in case of  $\alpha=1$  we talk about complete dependence, which means that the mean breaking force of every chain link ( $l_0$  length section) is the same (Figure 10). If  $0 < \alpha < 1$  then there is some kind of dependence, but not complete and if  $\alpha=0$  then there is complete independence, which means that the breaking forces of some chain links are independent of each other meaning that a small yarn segment can be followed by another of much larger or smaller strength.

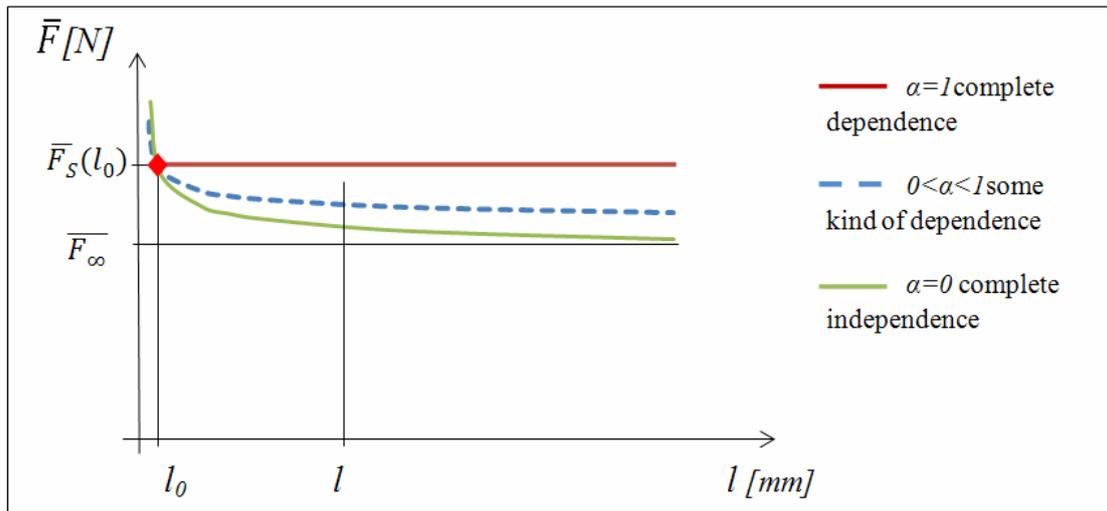


Figure 10 Influence of the convex linear combination

In our case  $\alpha=0.5$  was chosen and the linear combinations ( $\bar{F}(n \cdot l_0)$ ) denoted by *Peirce-Bott-Comb* and *Peirce-Upp-Comb* were calculated (Figure 11, dark and light blue solid line) creating a modified Peirce-zone. It can be seen that the Peirce-zone bounded by the obtained curves gives a good description of the range of the measured results.

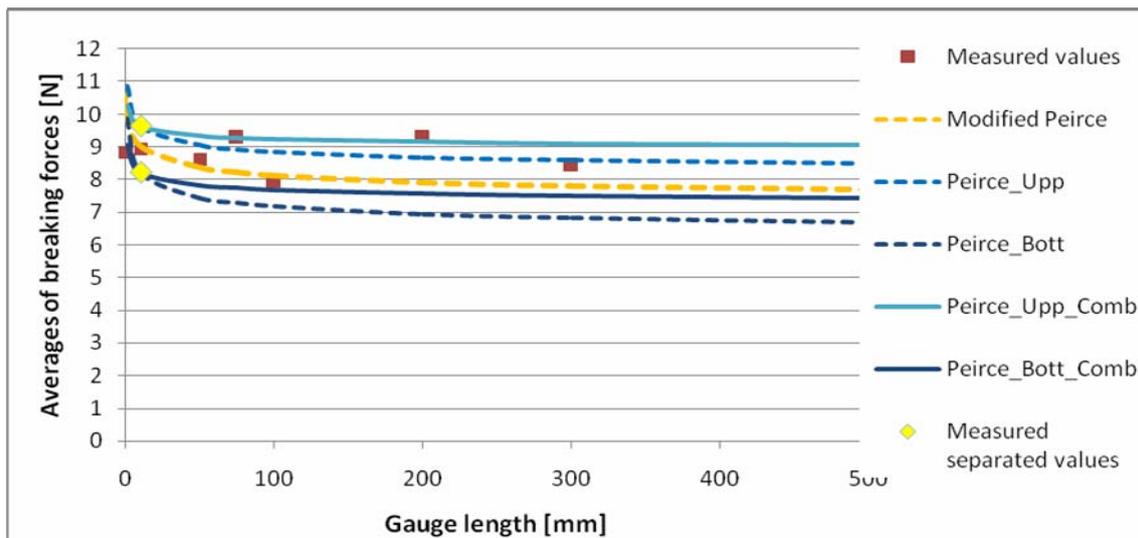


Figure 11 The modified Peirce-zone

#### 4 SUPPLEMENTARY ANALYSES

As a supplementary analysis further studies were done such as examinations of the twisted and untwisted parts and the utilization of the of the fibre strength.

##### 4.1 Examination of the twisted-untwisted parts

In order to reveal how the random localization of the twisted and untwisted yarn sections influence the results, further measurements were carried out. 25-25 tensile tests were performed at 10 mm gauge length, in a way that in one case the twisted part situated in the middle of the gripping section and the untwisted was in the grips, while in the other case it was inversely.

On the left side of Figure 12 the earlier measurements of random gripping are represented. In the column diagram the average of all the breaking forces and the mean of those which are below or above the mean value.

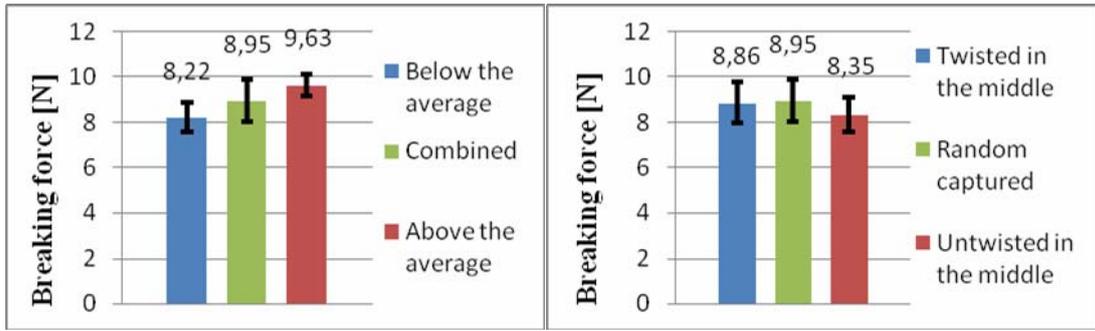


Figure 12 Comparison of the values below and above the average and twisted, untwisted in the middle

On the right side of Figure 12 the results of the examinations, which were carried out using the systematic gripping mode, are presented. The mean breaking force of the central untwisted sections corresponds to the below average results of random gripping. As regards the twisted sections in the middle, their average value is approximately equal to the overall mean obtained by random gripping. As for the mean of above average of random gripping it can be hypothesized that the higher breaking forces can be attributed to the more correct breaking and/or the cases of combined gripping.

Overall, the results of this study confirm that the tensile strength values depend on the method of gripping and the above-average and below-average groups are in a kind of relation with the gripping mode.

#### 4.2 The evaluation of the fibre strength utilization factor using the Peirce-models

In order to determine fiber strength utilization factor tensile tests of the elementary fibres were done at 20 mm gauge length. For the determination of strength utilization factor the following formula was used [6]:

$$0 < \eta_n = \frac{\overline{F_{S,yarn}(l_0)}}{n \cdot F_{S,fibre}(l_0)} \leq 1 \quad (17)$$

where  $n$  is the number of fibres in a yarn cross section,  $\eta_n$  is the strength utilization factor that is the ratio of the mean breaking forces of the yarn and  $n$  fibres determined at the  $l_0$  examination length.

The value of  $\eta_n$  can never reach 1 or 100% because that would mean that the fibres in the yarn are completely parallel to each other and the standard deviation of the breaking strain is 0 as well. This proved to be false based on the microscopic and mechanical examinations.

Since there was not any yarn measurement at 20 mm gauge length, therefore the average breaking forces belonging to the 20 mm yarn length were estimated with the help of the modified Peirce-models developed. Table 3 contains the values of the estimated mean breaking force and the strength utilization factors. The latter that ranges between 58% and 68% may be important for designing structures built up of this yarn.

Table 3 Fibre strength utilization factor

	Peirce_Up_Comb	Modified Peirce	Peirce_Bott_Comb
Gauge length [mm]	20	20	20
$F_{s\_fibre\_ave}$ [N]	0.15	0.15	0.15
Number of filaments, $n$ [-]	92	92	92
$F_{s\_yarn\_ave}$ [N]	9.5	8.8	8.1
$\eta_n$ [%]	68.4%	63.4%	58.4%

## 5 CONCLUSION

Structural and strength examinations of the yarn, which is the building element of the reinforcement of the flexible composite sheets planned to produce, were carried out. The electron microscopic recordings clearly represent the special periodically twisted and untwisted structure of the textured polyester multifilament yarn.

Tensile tests of the yarn were performed to determine the breaking force at different gauge lengths, which were compared with those estimated with the Peirce-models. The results fitted neither to the original, nor to the modified Peirce-model appropriately. With the development of the model, the bottom and upper values of the breaking force measured at the base length were separated and based on them a so called Peirce-zone bounded by bottom and upper curves was determined. Using the linear convex combination a modified Peirce-zone was created that covered well the measured data.

Finally, the fibre utilization factor of the yarn was determined that came about 58-68% giving important information for designing fabrics.

The research will be continued with the examination of the woven fabric as reinforcement and this fabric reinforced flexible composite sheet. Based on the results of these examinations a material model will be created for the computational design of products made of these composite sheet.

## 6 ACKNOWLEDGEMENT

This work was supported by the Hungarian Scientific Research Fund (OTKA, H), the National Development Agency (NDA, H), the National Centre for Scientific and Technical Research in the Kingdom of Morocco (CNRST) since this multinational study has been carried out commonly as part of the project K68438 (OTKA-NDA), as well as of the S&T project TÉT-10-1-2011-0493 (NDA) respectively. This project is supported by the New Széchenyi Plan (Project ID: TÁMOP-4.2.1/B-09/1/KMR-2010-0002) and connected to the scientific program of the "Development of quality-oriented and harmonized R+D+I strategy and functional model at BME" project.

## 7 REFERENCES

- [1] Picture available from: <http://hg.hu/cikk/epiteszet/3893-satrat-huznak-a-sportcsarnok-fole>
- [2] Picture available from: <http://www.olympia-leipzig-2012.de>
- [3] Hegyi D.: Nonlinear Analyses of Tent Structure and Technical Textile by Numerical and Experimental Methods (in Hungarian), PhD Thesis, BUTE Budapest (2006)
- [4] Peirce F.T.: The Weakest Link, Theorem on the Strength of Long and Composite Specimens. *Journal of The Textile Institute Transactions* 17(7) T355-T368 (1926)
- [5] Vas L. M., Halász G.: Relationships between Structure and Strength Properties by Different Yarn Structures Parts II-III, (in Hungarian) *Magyar Textiltechnika*, Part II: 47(6) 192-201 (1994), Part III: 48(1), 1-4 (1995)
- [6] Vas L.M., Tamás P.: Modelling Size Effects of Fibrous Materials Using Fibre-Bundle-Cells. ECCM-14 14th European Conference on Composite Materials. Budapest, 7-10 June, 2010. Proceedings Paper ID-705, 1-11. ISBN: 978-963-313-008-7.

### Corresponding author:

Alexandra Csilla KOVÁCS  
Department of Polymer Engineering  
Faculty of Mechanical Engineering  
Budapest University of Technology and Economics  
H-1111 Budapest, Műegyetem rkp. 3, Hungary  
Phone: 06-70-331-11-41, Fax: 06-1-463-1527  
E-mail: kov.ale29@gmail.com