FIBER-BUNDLE-CELLS MODEL OF COMPOSITES

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Abstract

In this paper a fiber-bundle-cells model based method is presented for describing and analyzing the tensile behavior of textile fabrics reinforced composites in main directions and its applicability is demonstrated in the case of fabric reinforced membrane. As an application aramid fabric reinforced PVC membrane specimens were tested in cases of 50 and 100 mm gauge lengths. From their E-bundle based analysis the statistical parameters of the model composite elements and the parameters of transforming the mean force-elongation curve determined at 50 mm gauge length into that at 100 mm gauge length. The obtained parameter values could be explained and related to observations.

Introduction

The fibrous structures such as textile materials, the fiber reinforced composites, and the linear polymers, too, are built up of discrete fiber-like elements, textile or reinforcing fibers, or yarns as e.g. the element of fabrics. The adjoining fibers or those intersecting a cross section of a fibrous sample create certain small assemblies that are fiber bundles in which the fibers show collective group-behavior [1-3]. The fiber bundle can be treated as intermediate elements of a fibrous structure which can represent the statistical properties of the geometry or the strength. Besides the classic one [1-3], L.M. Vas et al. [4-9] have introduced some other idealized statistical fiber bundles called fiber-bundle-cells (FBC) and shown that they have been able to be applied to modeling some structural and strength properties of fibrous materials.

In this paper an FBC model based method is presented for describing and analyzing the tensile behavior of textile fabrics reinforced composites in main directions and its applicability is demonstrated in the case of fabric reinforced membrane.

Fiber bundle cells based modeling method

Statistical fiber bundle cells

Fiber bundles are defined as fiber classes containing the same geometrical (shape, disposition) and mechanical properties (strain state, gripping by the environment). These fiber classes are called fiber bundle cells (FBCs) (Fig. 1) [4, 6]. Fibers of these FBCs are supposed to be perfectly flexible, linearly elastic and to break at a random strain (ε_S). They are straight in the E-bundle, loose (ε_0<0) or pre-tensioned (ε_0>0) in the EH-bundle, and oblique (fiber angle $\beta \neq 0$) in the ET-bundle, and gripped ideally in these cases. Fibers in the ES-bundle are straight but they may slip out of their grip at a strain level ($\varepsilon_b < \varepsilon_S$) or create fiber-chains with slipping bonds. Both the shape, position, and strength parameters of fibers are assumed to be independent stochastic variables.

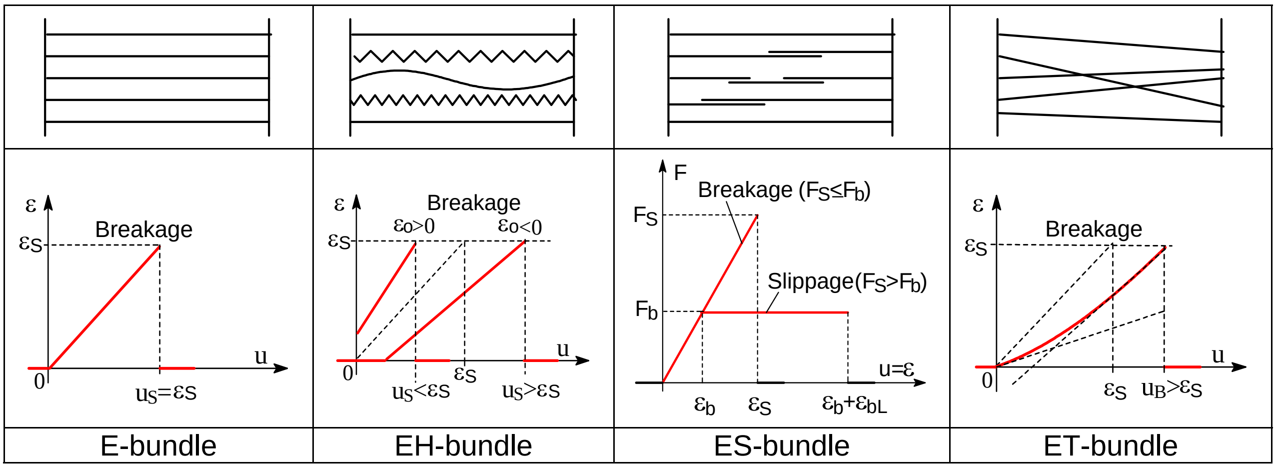


Fig. 1 Structural scheme of the idealized fiber bundle cells and relationship between the strains of single fibers and FBCs

Considering a constant rate elongation tensile test the tensile force ($F(u)$) creates a stochastic process as a function of the bundle strain (u). Being aware of the relationship between the bundle (u) and fiber strains (ε), the expected value of the tensile force of the FBCs ($E(F) = \bar{F}$) could be calculated as a sum of the single fiber forces using the suitable formulas developed [4, 6]. Dividing the expected value by the mean breaking force of fibers, the normalized tensile force of bundle is computed as follows:

$$0 < FH(z) = \bar{F}(z)/n\bar{F}_S \le 1, \qquad z = u/\bar{\varepsilon}_S \tag{1}$$

where n, $\bar{F}_S$, and $\bar{\varepsilon}_S$ are the number, the mean breaking force and strain of fibers, respectively, and z is the bundle strain normalized by the mean breaking strain of fibers. Fig. 1 shows the graphic relationship between the strain of individual flexible fibers and the bundle, as well. In case of ES-bundle ε_{bL} is the relative slippage way of fibers.

Parallel and serial connection of FBCs

In Fig. 2.a the typical normalized expected value processes calculated at different parameter values are plotted for the FBCs [6, 8]. For the numerical calculations all random parameters were assumed to be of normal distribution. From Fig. 2.a it is obvious that the FBCs can model rather complicated mechanical behaviors such as the initial convex part caused by crimped fibers (EH-bundle) or the slippages generated plateau beyond the peak (ES-bundle) even if they are used in themselves.

In general, several types of FBCs are needed to model the response of a real fibrous structure. In most cases the parallel connected FBCs (Fig. 2.a) called composite bundle provide a suitable model and the resultant expected value process is calculated as the weighted sum of the single FBC responses where the weights are the fiber number ratios [6, 8]. In case the size effect such as the gauge length on strength are examined serial connection of the same type of independent FBCs is suitable to use creating a bundle chain (Fig. 2.b) [6, 8]. As examples Fig. 2.a shows the weighted sum of the normalized force-strain curves (percentages are the relative weight values of FBCs) while in Fig. 2.b the effect of the number (m) of serial connected independent E-bundles is demonstrated causing the decrease of the peak value of the resultant force-strain curves that characterizes the tensile strength of the E-bundle chain (VE is the relative standard deviation of ε_S).

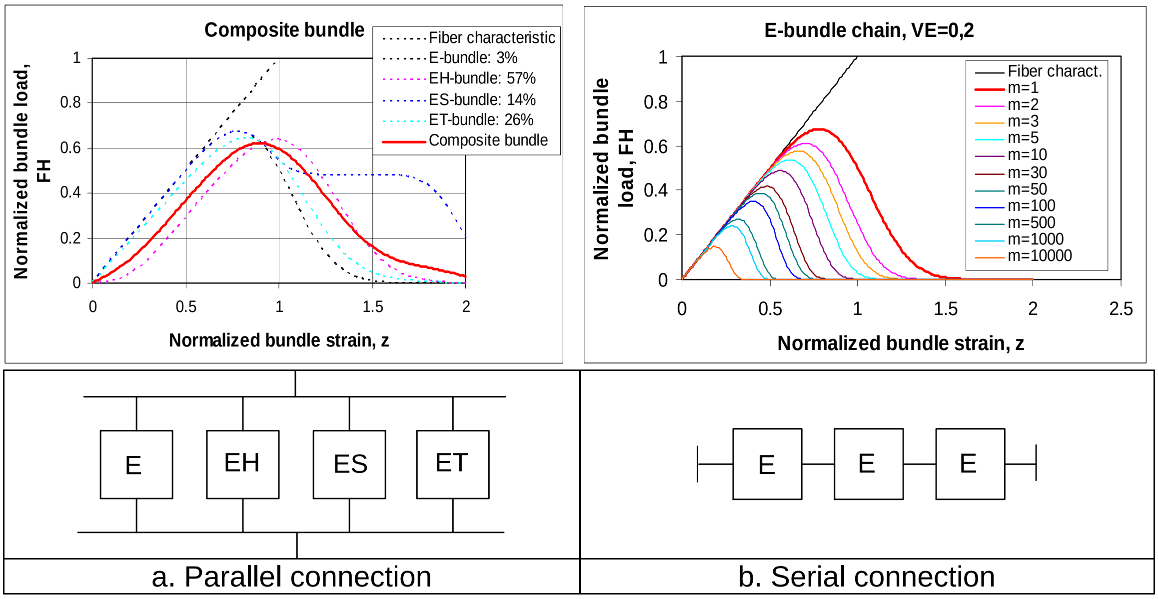


Fig. 2 Schemes and normalized mean force-strain curves of parallel (a) and serial (b) connected FBCs

FBC modeling of fabric reinforced composites

In the case of the fabric reinforced composite the role of the fiber elements play the yarns. In this paper for modeling and analyzing the tensile behavior of samples cut out in the main structural directions of a simplified composite FBC modeling method is used neglecting the crimping of yarns as it is in the usual finite element layer models [10]. In this case, as a first step, E-bundles can be applied to modeling the reinforcement because the yarns in main directions are parallel to the tensile load and gripped at both ends creating an E-bundle approximately (Fig. 3.a). In the usual composite model the so called mixture rule is employed where, in the fiber direction, the fibers and matrix are treated as serial connected parts which work together until damaging.

Cutting out a sample e.g. in warp direction from the composite its reinforcement is built of warp yarns aligned lengthwise (Fig. 3.a) and weft yarns aligned crosswise at a distance δ (Fig. 3.b). Loading this sample in lengthwise direction the load is taken up by the warp yarns gripped at both ends and the weft yarns with free ends play just a modifying role by interlacing and crimping the warp yarns even if densely and so does the matrix. In general, the unidirectional composite in fiber direction is considered as a matrix embedded E-bundle chain forming by serial connected short (and approximately independent) matrix embedded E-bundles of yarns (Fig. 3.c). The length of these bundles (l_o) is the so called ineffective length (or critical adhesion length) that may be different of δ because of the matrix adhesion and the possible slippage of the weft yarns [1, 3]. This matrix embedded bundle can be treated as a special bundle the elements of which are matrix embedded yarns.

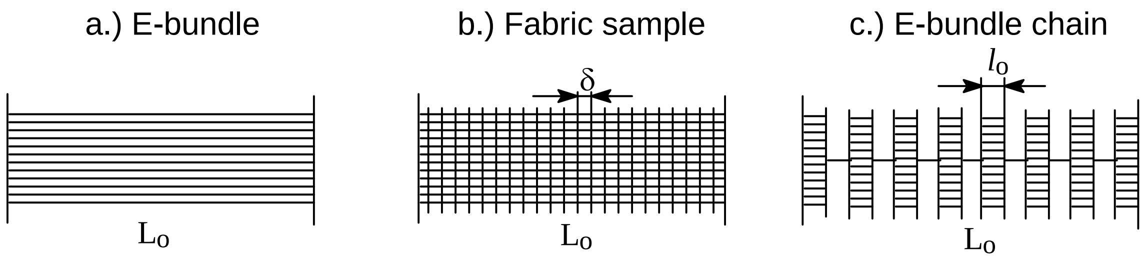


Fig. 3 E-bundle (a), a fabric sample cut out in main direction (b) and E-bundle chain (c) as the model of the sample

The expected value of the tensile force process of the E-bundle (Fig. 1) can be calculated by the following formula [4, 6]:

$$\overline{F}_y(u) = E[F_y(u)] = \overline{K}_y f_y(u)\left(1 - Q_{\varepsilon_{Sy}}(u)\right) \tag{2}$$

where u is the bundle strain, $\overline{K}_y$ and $Q_{\varepsilon sy}$ are the collective mean tensile stiffness and the distribution functions of the yarn breaking strain (ε_{Sy}) of the yarns, respectively. $f_y(u)=ug_y(u)$, $g_y(0)=1$, is the normalized tensile characteristic of the yarns which is in simple cases linear that is $g_y(u)\equiv 1$.

A single composite elements described above consists of a warp yarn (or a segment of that) and its matrix vicinity forming a kind of core-sheath structure, consequently, the matrix sheath elements can be considered as an E-bundle as well, the mean force process of which is as follows:

$$\overline{F}_m(u) = E[F_m(u)] = \overline{K}_m f_m(u)\left(1 - Q_{\varepsilon_{Sm}}(u)\right) \tag{3}$$

where the parameters and variables are to be understood as those for the yarns. Using the so called mixture rule the resultant tensile force of the composite element is the sum of those in the yarn and the matrix (Fig. 4.a):

$$\overline{F}(u) = \overline{F}_y(u) + \overline{F}_m(u) = \overline{K}_y f_y(u)\left(1 - Q_{\varepsilon_{Sy}}(u)\right) + \overline{K}_m f_m(u)\left(1 - Q_{\varepsilon_{Sm}}(u)\right) \tag{4}$$

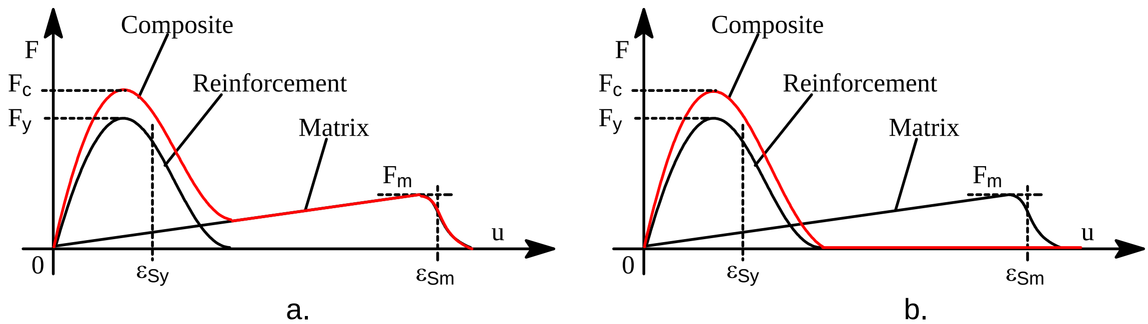


Fig. 4. Tensile force processes of yarn, matrix and composite in cases of independent failures (a) and when the yarn breakages dominate the failure process of the composite

Usually the breaking strain of the matrix much greater than that of the reinforcing yarns and the breakage of a yarn causes the crack and failure the matrix in its vicinity consequently the yarn breakages control the failure of the composite (Fig. 4.b). Supposing this phenomenon the resultant mean tensile force process of the composite sample can be approximated with the following formula:

$$\overline{F}(u)=\overline{F}_y(u)+\overline{F}_m(u)=\left(\overline{K}_y f_y(u)+\overline{K}_m f_m(u)\right)\left(1-Q_{\varepsilon_{Sy}}(u)\right)=\kappa(u)R(u) \tag{5}$$

where the mean tensile characteristic (κ) and tensile stiffness (K) and the reliability function (R) of the composite are respectively:

$$\kappa(u)=\overline{K}_y f_y(u)+\overline{K}_m f_m(u)=K\left(\frac{\overline{K}_y}{K} f_y(u)+\frac{\overline{K}_m}{K} f_m(u)\right) \tag{6}$$

$$K=\overline{K}_y+\overline{K}_m \tag{7}$$

$$R(u)=1-Q_{\varepsilon_{Sy}}(u) \tag{8}$$

The yarn and matrix parameters can be determined by tensile tests of yarn samples of gauge length L_o. All that is valid for gauge length L_o at which the tensile test is performed and it is well known that the tensile strength parameters of yarns depend on the gauge length that is the mean breaking force of fibers and yarns and its standard deviation increase with reducing the gauge length which is known as size effect in the literature [1-3, 11]. In the subsequent section these formulae will be applied with using absolute elongation (λ) instead of strain because of the additivity of the former.

Supposing the gauge length is changed for $L=mL_o$ (n=1,2,…) and the sections of composite elements of length L_o create a E-bundle chain of independent elements (Fig. 2.b) then the expected tensile process of an E-bundle created by composite elements of length L can be calculated from that for the bundle of length L_o as follows [9] (Fig. 2.b):

$$\overline{F}(\lambda_o;L_o)=\kappa_o(\lambda_o;L_o)R_o(\lambda_o;L_o) \tag{9}$$

$$\overline{F}(\lambda;mL_o)=\kappa(\lambda;mL_o)R(\lambda;mL_o)=\kappa_o(m\lambda_o;L_o)R_o^m(m\lambda_o;L_o) \tag{10}$$

where λ_o and $\lambda=m\lambda_o$ are the elongation of the E-bundles of lengths L_o and mL_o.

The formulae discussed above can be used for taking into account the size effect in the reinforcing structure caused by changes in effective length. It can be shown that this bundle chain model corresponds to Peirce's "weakest link" principle [1-3].

Application of the FBC model

Experimental

To demonstrate the applicability of the FBC model of fabric reinforced composites samples cut out of a plain woven aramid fabric reinforced PVC membrane (B8000 Sioen) were tested (Table 1) (the yarn density is measured).

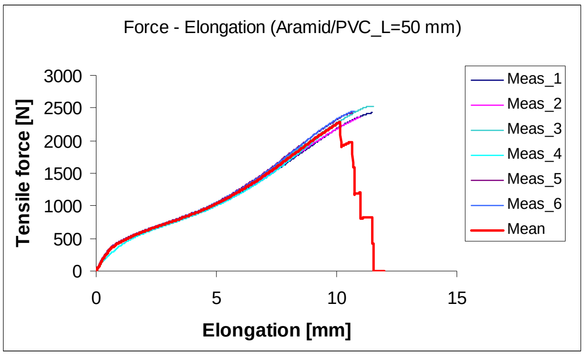
Yarn			**Fabric**			**Composite**	
Material	**Linear density** [dtex]		**Type of weave**	**Yarn density** [1/100 mm]		**Matrix material**	**Density** [g/m^2]
	warp	weft		warp	weft		
Aramid	1100		plain	75		PVC	630

Table 1 Data of composite membrane examined

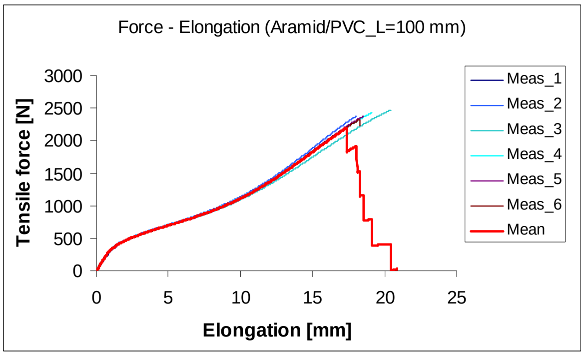
The width and length of the samples were 50 mm and 200 mm, respectively. The tensile tests were performed on tester Zwick Z50, the test speed was 10 mm/min, and the gauge lengths were 50 and 100 mm. The results are summarized in Table 2. The recorded force elongation curves and their average calculated point by point can be seen in Fig. 5.

Measurements		**A_L=50**	**A_L=100**
Mean Fmax	[N]	2412	2362
SD_F	[N]	82.2	92.4
Rel. SD_F	[%]	3.41	3.91
Mean E_Fmax	[mm]	10.88	18.64
SD_E	[mm]	0.52	1.06
Rel. SD_E	[%]	4.79	5.67
Infl_Stiffn.	[N/mm]	280	168

Table 2 Tensile test result of specimens (SD – standard deviation, E_Fmax – elongation belonging to F_{max}, Inf_Stiffn. – tensile stiffness at the onset inflexion point of the force-elongation curve)



a.



b.

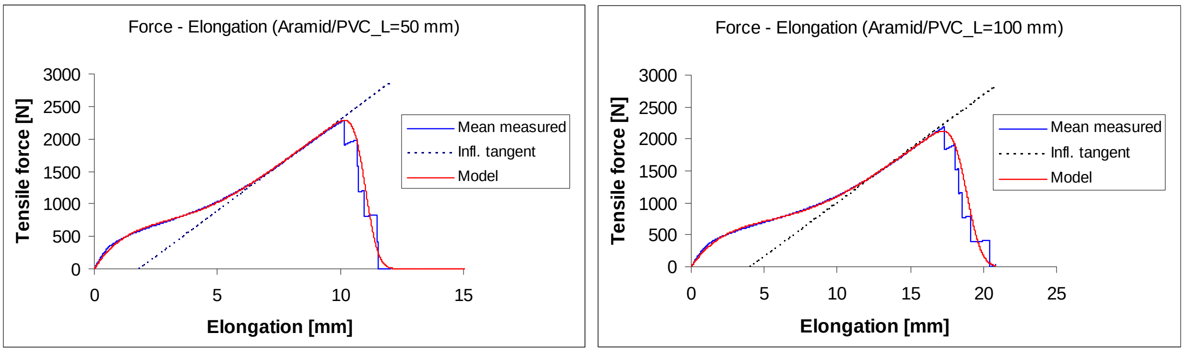
Fig. 5 Tensile test measurements performed at gauge lengths 50 mm (a) and 100mm (b) (Mean – average curve)

From the shape of the force-elongation curves it is obvious that the yarns are of non-linear tensile characteristic and so are the composite elements.

Results of FBC modeling and their analysis

In order to create a simple global FBC model on the basis of formula (5), using absolute elongation instead of strain, firstly the tensile characteristic of the composite is to be determined. It was approximated by the measured force-elongation curve up to the onset inflexion point and then the inflexion tangent and a polynomial of the 5^{th} degree was fitted. Using this polynomial as tensile characteristic κ in Equation (5) the reliability function of the composite was determined by fitting that to the falling part of the average curve supposing the breaking strain of the damaging composite elements are of normal distribution.

The graphical results of these calculations are depicted in Fig. 6, the model parameters are summarized in Table 3.



a. b.

Fig. 6 The averaged force-elongation curve of 6 measurements performed at gauge lengths 50 mm (a) and 100mm (b) and the model-curves

From comparing Tables 2 and 3 it can be stated that for both gauge lengths the mean breaking elongations of the composite elements are a little greater than those of the elongations belonging to the maximum forces, while the standard deviations of the model elements are significantly smaller. This means that the peak elongations have significantly greater uncertainty that is being aware of the model parameters makes possible to do more accurate calculations. There is a good agreement between the measured modeled mean peak force values. On the other hand, if the breaking elongation distribution of the real yarns and the fabric are known then further analysis can be carried out concerning the mechanical behavior of the composite structure.

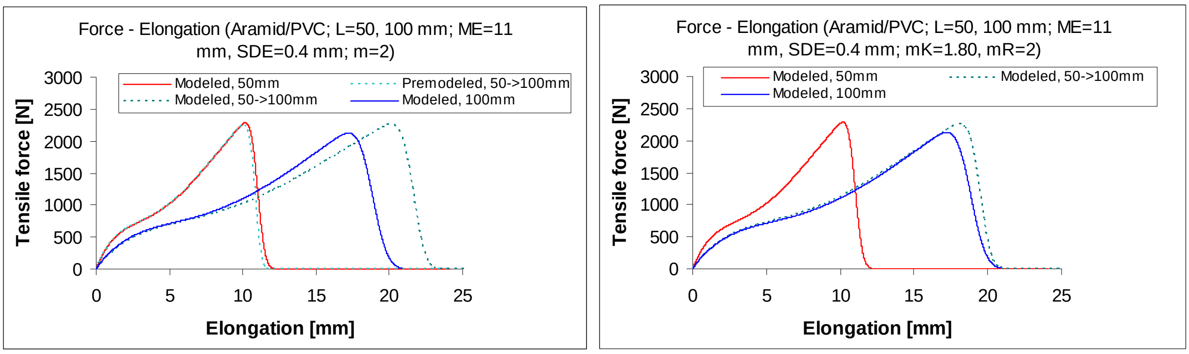
Model		A_L=50	A_L=100
Mean c_Fmax	[N]	2289	2195
Model_McE	[mm]	11.00	18.80
Model_SDcE	[mm]	0.40	0.80
Model_Fmax	[N]	2288	2127

Table 3 Results of FBC modeling using yarns of 50 mm length and measurements (mean) (Mean c_Fmax – maximum force of the averaged curve, McE and SDcE are the mean and standard deviation the model composite elements)

Making use of Equations (9) and (10) the force-strain curve measured at shorter gauge length (50 mm) or its FBC model can be transformed into that for the longer one (100 mm). In the first step a linear variable transformation is carried out on the model curve for 50 mm gauge length in the case of m=2 that is the ratio of the two gauge lengths (Fig. 7.a). The transformed curve fits well to the blue one up to 7 mm elongation but afterward it produces larger elongation at the same force levels indicating some difference in structural-mechanical changes between the specimens of different gauge lengths.

Changing the scale factor in the tensile characteristic from 2 to 1.80 and holding the exponent of the reliability function (m=mR=2) results in good fitting along the increasing part but it is not proper in the decreasing one (Fig. 7.b).

The tensile characteristic describes the functioning without any damage or failure while the reliability function takes the damage process into account. Therefore it is acceptable if the scale factor and the exponent defined above are different in real structures. Moreover, the new scale factor (1.80) is close to the ratio of the mean elongations belonging to the measured peak forces (Mean E_Fmax in Table 2) which is 1.71.



a. b.

Fig. 7 Model force-elongation curve for 50 mm and 100 mm gauge lengths (red and blue lines) and the transformed form of the former (green dotted lines)

In order to improve the fitting the distribution parameters of the breaking elongation of the composite element and the exponent of the reliability function were changed as well. The best fitting (Fig. 8) was obtained at the parameter values in Table 4. This operation claimed increasing the mean breaking elongation a little (5.5%), the standard deviation considerably (80%), and the exponent strongly (6 times) (Table 4). Based on these results some structural-mechanical analysis can be carried out.

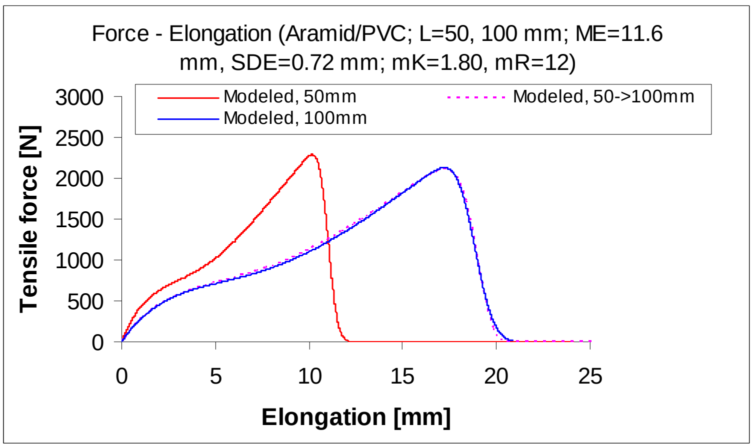


Fig. 8 Modeled force-elongation curves (red and blue lines) for gauge lengths 50 and 100 mm, and the transformed model curve (dotted lilac line)

Modelparameters	**Aramid/PVC_50-->100**		
Model_McE [mm]	11.00		11.60
Model:SDcE [mm]	0.40		0.72
mK [-]	2.0	1.80	1.80
mR [-]	2.0	2.0	12.0
Model_Fmax [N]	2259	2259	2106

Table 4 Results of transforming the force-elongation model curve for L=50 mm into that for L=100 mm

Exponent '*m*' means the number of elements in a bundel chain where the bundle element corresponds to the shorter sample of length L_o. L_o can be related to the length of the damaging zone where the ultimate failure takes place. Therefore the exponent (m=12) obtained by optimal transformation according to Equations (9) and (10) can be understood that the specimen of 100 mm gauge length has got a damaging zone 12 times larger than that of the specimen of 50 mm gauge length. After all this seems to be too large if one considers the usual cases. On the other hand the standard deviation of the breaking elongation of the composite model elements came about much greater than that for 50 mm length. This should be inversely if considering the results of the ‘weakest link’ principle [1-3]. It is obvious that the larger standard deviation should be compensated by larger exponent.

All that can be explained by that the specimens of 200 mm length could be clamped lighter and more accurately in case of 50 mm gauge length than it was 100 mm because in the previous case tha specimen reaches beyond the grips by 50-50 mm. Consequently the uneven gripping caused uneven stress distribution along the cross section of the specimen which increased the the length of damaging zone and by that the mean and standard deviation of the breaking elongation. This was supported by observing the place of breakages. In each case at 50 mm gauge length the breakage went along a cross section between two weft yarns meaning that the length of the breaking zone was about 100/75=1.33 mm (Table 1) measured after testing in stressless state of the specimens. The specimens tested at 100 mm gauge length showed another breakage image. The path of yarn breakages went in a direction oblique to the cross section and created a kind of fractal. The length of the breaking zone varied between 2.5 and 9.5 mm meaning that it was about 2-8 times larger than that in case of 50 mm length and the larger zone range is in good agreement with the larger standard deviation of the breaking elongation of the model component elements.

Summary

An E-bundle based evaluation method of the tensile strip test of fabric reinforced composite sheets in the main directions was developed for modeling the mean force-elongation curve at different gauge length and analyzing the structural-mechanical behavior of specimens tested and the effect of the gauge length.

As an application aramid fabric reinforced PVC membrane specimens was tested in cases of 50 and 100 mm gauge lengths. From their E-bundle based analysis the statistical parameters of the model composite elements and the parameters of transforming the mean force-elongation curve determined at 50 mm gauge length into that at 100 mm gauge length. The obtained parameter values could be explained and related to observations.

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