14TH EUROPEAN CONFERENCE ON COMPOSITE MATERIALS
7-10 June 2010, Budapest, Hungary
Paper ID: 648-ECCM14

TESTING AND MODELLING THE BENDING BEHAVIOUR OF FLEXIBLE COMPOSITE SHEETS

B. Al-Gaadi [1a], M. Halász [1b], L. M. Vas [1c], P. Tamás[2]

Budapest University of Technology and Economics, Műegyetem rkp. 3., H-1111, Budapest, Hungary
[1]Department of Polymer Engineering, e-mail: [1a]al-gaadi@pt.bme.hu, [1b]halaszm@pt.bme.hu, [1c]vas@pt.bme.hu
[2]Department of Mechatronics, Optics and Engineering Informatics, e-mail: tamas@mogi.bme.hu

Abstract

In a lot of applications composite sheets with great flexibility are desired to take up different 3D shapes. Flexible composite sheets are used for example for roofs or tents in the architecture, tanks or containers in the transport industry, etc. To examine the flexibility of composite sheets we measured their bending rigidity with optical bending test equipment developed at the Department of Polymer Engineering, Budapest University of Technology and Economics. The equipment has an advantage opposite to traditional ones namely the measuring process works without mechanical contact. There are three laser lines projected onto the bended material that are stored on photos taken with a camera. Evaluating the measurement makes it possible to determine the bending stiffness and other bending properties of the flexible composite sheet specimen from the deflection curves or surface created by image processing and mathematical methods. The results of measurements can be used for modelling the bending behaviour of the composite sheets of more complicated shapes. With the obtained model we can predict the behaviour of composite sheets during their application.

1 Introduction

Simulating the drapability behavior of textiles basically requires the knowledge of their bending properties. The bending behavior of textiles is mainly characterized by the bending length measured by Cantilever Test developed by Peirce [1]. Another characteristic is the bending stiffness measured by the special bending tester instrument of KES-F system [2].

We present a measurement method working in a new optical way – similar to our formerly developed system for measuring fabric drape [3] – which makes it possible to determine the bending characteristic of textiles with application of image processing.

During the measurement the fabric sample is gripped on its two ends and three laser lights are projected onto the sample deflecting freely between the two grips. The laser curves on the bended sample are recorded with a camera and their shape is determined by image processing making it possible to create a virtual surface of the bent sample. The bending characteristics and the data such as bending stiffness needed to simulate drape behavior are determined by mathematical methods from the determined shape.

In this paper the instrument developed for measuring bending characteristics, and the way of measurement and evaluation are presented.

2 Measuring Instrument

The test table of the equipment consists of several element pairs (upper and lower sections) of the same dimensions. Upper sections can be turned up separately and the lowers can be turned down separately, as well. During measurement a 100 mm x 600 mm specimen is laid on the turned down lower elements without strain and is fixed by turning down the upper elements. There is a non-slippery coating on the fixing elements. Due to the construction the bending effect can be measured at different lengths. During the measurement a suitable number of the upper fixed parts are turned up and the lower ones opposite to them are turned down. The specimen will deflect downwards because of gravity. There is a bijection between the mechanical parameters and the shape of the specimen. There are three laser projectors above the table and a camera. The camera takes the image of the projected laser lines (Figure 1.).

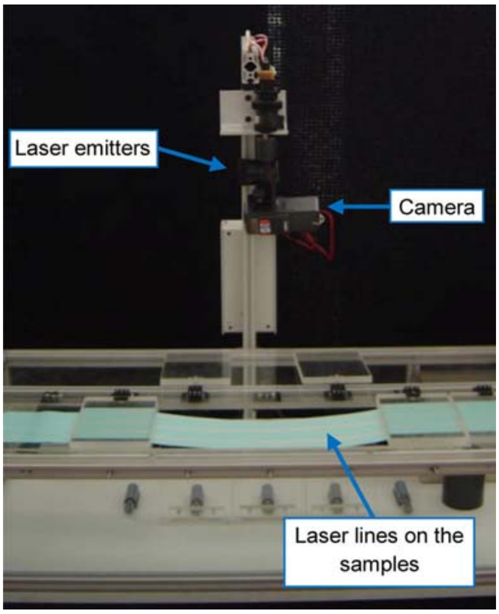

Figure 1. Instrument for measuring bending properties of textiles

3 Calibration and Image Processing

In order to develop suitable measuring methods and achieve sufficient accuracy, the cross-sections in the photo had to be calibrated and errors had to be analyzed.

Light-beams form planar curves in three positions. The points of curves are determined by image processing. For 3D scanning the plane to plane perspective transformation is a bijection. Perspective transformation with homogenous coordinates is a linear transformation [4] that projects quadrangle to quadrangle. The matrix of transformation (1) has eight independent coordinates.

$$\underline{\underline{P}} = \begin{bmatrix} p_0 & p_1 & p_2 \\ p_3 & p_4 & p_5 \\ p_6 & p_7 & 1 \end{bmatrix} \quad (1)$$

Corners of a rectangular calibration element are appropriate to define the matrix coordinates (Figure 2).

Corners of calibration equipment are (t_x^i, t_y^i), and corners of its picture are (v_x^i, v_y^i) (i=0, 1, 2, 3) and the transformation is shown in Equation (2).

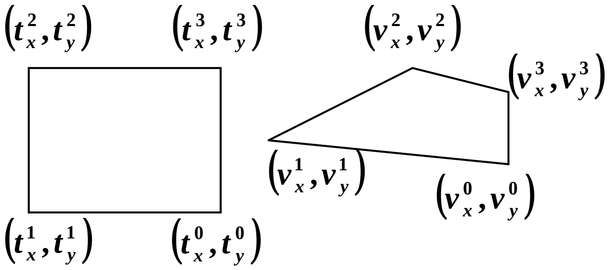

Figure 2. Planar perspective projection

$$\begin{bmatrix} v_x^i \\ v_y^i \\ 1 \end{bmatrix} = \begin{bmatrix} p_0 & p_1 & p_2 \\ p_3 & p_4 & p_5 \\ p_6 & p_7 & 1 \end{bmatrix} \cdot \begin{bmatrix} t_x^i \\ t_y^i \\ 1 \end{bmatrix} \tag{2}$$

There are eight unknown coordinates and eight equations represented in formulas (3) in every plane [5].

$$v_x^i = \frac{p_0 \cdot t_x^i + p_1 \cdot t_y^i + p_2}{p_6 \cdot t_x^i + p_7 \cdot t_y^i + 1} \qquad v_y^i = \frac{p_3 \cdot t_x^i + p_4 \cdot t_y^i + p_5}{p_6 \cdot t_x^i + p_7 \cdot t_y^i + 1} \qquad i = 0, 1, 2, 3 \tag{3}$$

Determination of corner coordinates can be defined from the calibration photo (Figure 3).

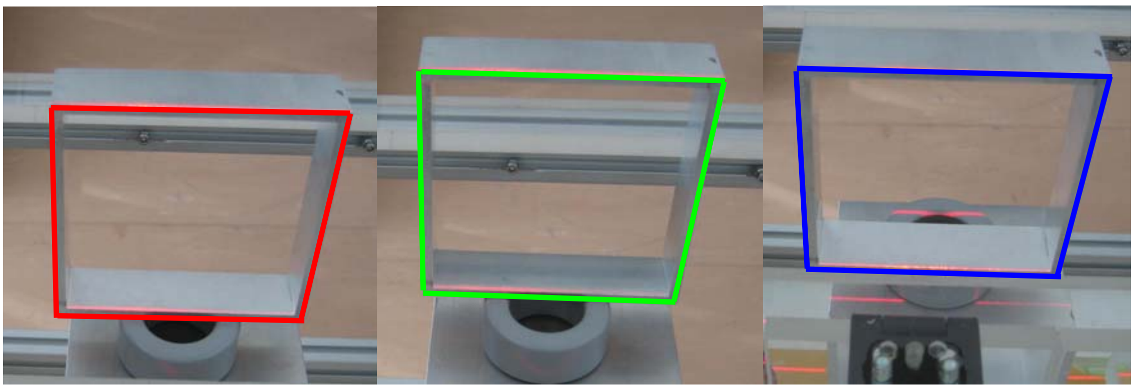

Figure 3. Calibrating rectangles

Different curves of beams can be processed from the picture (Figure 4). [6].

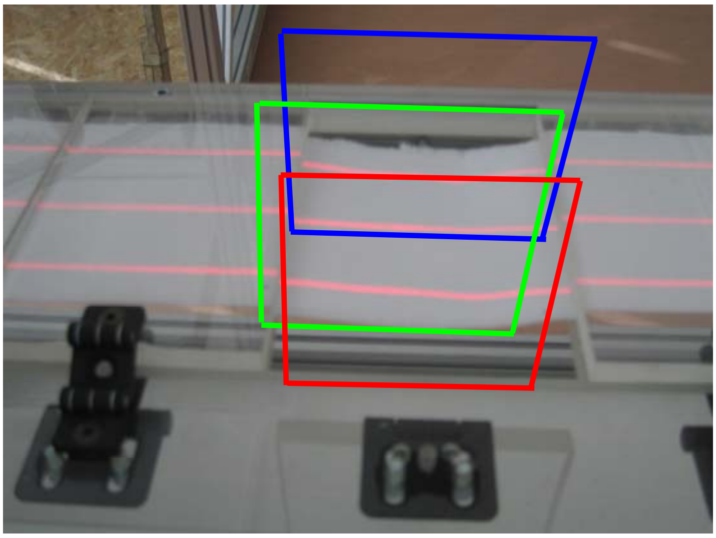

Figure 4. Measuring Process

4 Estimation of bending parameters

The strip-like specimen cut out of fabrics, fibrous mats, or flexible fiber reinforced composites is laid on a horizontal plate and gripped at both ends at span length L_o. After removing the supporting plate between the grips the specimen stretches under its weight and takes up a curved shape that is represented by the middle line in Figure 6. This middle line can be determined as a y(x) function from the image of the laser beams projected on the specimen using the developed suitable image processing methods. Before loading the initial geometrical parameters of the specimen of rectangular cross section are the cross section area ($A_o=b_oh_o$), the inertial moment ($I_o=h_o^3b_o/12$), and the linear density ($q_o=\rho A_o$) where b_o and h_o are the width and the thickness, respectively, and ρ is the volume density.

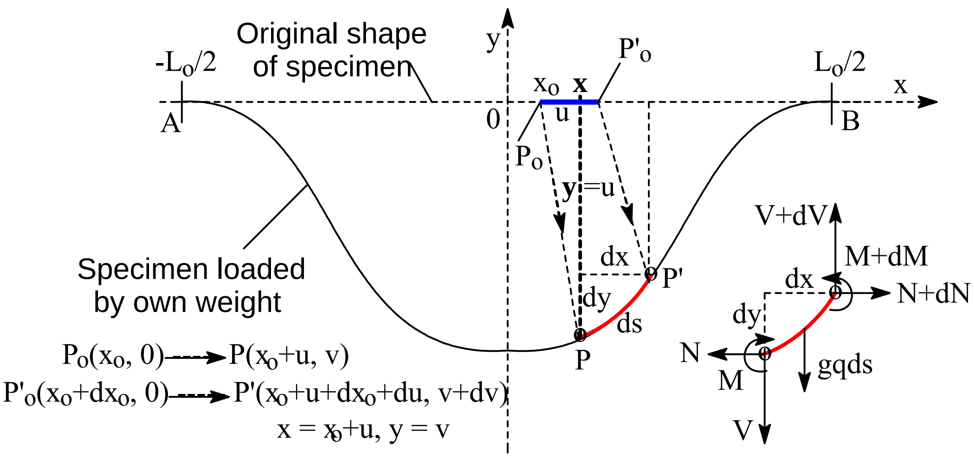

Figure 6. Shape of a fabric specimen and the displacement and deformation of a small segment due to forces and bending moments

Supposing that the material of the specimen is linearly elastic with finite tensile (A_oE) and bending (I_oE') stiffness – E and E' are the tensile and bending modulus, respectively – the equilibriums of forces and moments on a length segment ds provide the following equations (Figure 6):

$$N = N_o = constant; \quad \frac{dV}{dx} = gq\sqrt{1+y'^2}; \quad \frac{dM}{dx} = Ny' - V \tag{4}$$

where q is the local linear density of the loaded specimen, y' and y" are derivatives, g is the gravitational acceleration. The relationship between the bending moment and the local curvature (κ) is well known from the classical bending theory [7-10]:

$$M = IE'\kappa; \quad \kappa = \frac{y''}{\left(1+y'^2\right)^{3/2}} \tag{5}$$

where I is the inertial moment of the cross section of the loaded specimen. For wide specimens, if $b_o > 6h_o$, E' is to be calculated with the Young modulus (E) and the Poisson factor (ν) as follows [8]:

$$E' = \begin{cases} E, & b_o \le 6h_o \\ \dfrac{E}{1-\nu^2}, & b_o > 6h_o \end{cases} \tag{6}$$

The crosswise contraction is taken into account with assuming volume permanence during deformation hence the cross section (A), the linear density (q), and the inertial moment (I) of the loaded specimen are as follows:

$$A = \frac{A_o}{1+\varepsilon}, \quad q = \frac{q_o}{1+\varepsilon}, \quad I = \frac{I_o}{(1+\varepsilon)^2} \tag{7}$$

Combining all the equations obtained above the following differential equation can be derived:

$$I_o E' \frac{d^2}{dx^2}\left[\frac{\kappa(x)}{(1+\varepsilon(x))^2}\right] = N_o y'' - gq_o \frac{\sqrt{1+y'^2}}{1+\varepsilon(x)} \tag{8}$$

In equation (8) the unknown parameters are E' and N_o. They can be determined if the deflection curve, y(x), and the local strain ε(x) are known. To the latter Hooke's law can be applied:

$$F(x) = AE\varepsilon(x) = A_o E \frac{\varepsilon(x)}{1+\varepsilon(x)} \tag{9}$$

where F(x) is the tangent projection of the resultant force (Figure 7), for which the following equation is obtained in a way that tgα=y' is taken into account, as well:

$$F(x)=\sqrt{N^2+V^2}\,cos(\beta-\alpha)=N_o\frac{1+\frac{V(x)}{N_o}}{\sqrt{1+y'^2}} \tag{10}$$

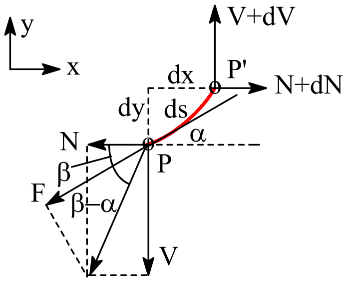

Figure 7. Tensile force (F) as the projection of the resultant in tangential direction

By combining equations (4), (9), and (10) the local strain, ε(x), can be expressed with the derivatives of y(x).
Using some simplifications described below the creation and use of another differential equation for ε(x) can be avoided besides retaining the effects of bending and stretching.

(1) Neglecting displacement u(x) ($u\equiv0 \Rightarrow x=x_o \Rightarrow 1+\varepsilon=\sqrt{1+y'^2}$) leads to the following differential equation:

$$I_oE'\frac{d^2}{dx^2}\left[\frac{\kappa(x)}{1+y'^2}\right]=N_oy''-gq_o \tag{11}$$

(2) Considering the mean strain ($\bar{\varepsilon}$) instead of the local one (ε) in equation (8):

$$\bar{\varepsilon}=\frac{s(L_o/2)-L_o/2}{L_o/2}=\frac{2}{L_o}\int_o^{L_o/2}\sqrt{1+y'^2}\,dx-1 \tag{12}$$

where s(x) is the arc-length of the loaded specimen.

(3) Neglecting the crosswise contraction (ε<<1) strongly simplifies equation (8):

$$I_oE'\frac{d^2\kappa(x)}{dx^2}=N_oy''-g\bar{q}\sqrt{1+y'^2} \tag{13}$$

where $\bar{q}$ is the mean linear density of the loaded specimen:

$$\bar{q}=q_o\frac{s(L_o/2)}{L_o/2}=\frac{2q_o}{L_o}\int_o^{L_o/2}\sqrt{1+y'^2}\,dx \tag{14}$$

Substituting the measured deflection function, y(x), into any of the simplified differential equations and considering that at the maximum deflection (x=0, y'(0)=0) and at any other point e.g. at the inflection point ($x=x_I$, $y''(x_I)=0$) or at the grip point (x=Lo/2, y'(0)=0) they provide two simply computable algebraic equations from which the sought parameters can be determined.

6 Results

The applicability of the measurement method on b=100 mm wide cotton fabric at L_o=300 mm grip length, i.e. span length was examined. The structural and geometrical data of the sample are summarized in Table 1.

Material	Type		Thickness [mm]	Density [g/m^2]	Type of weave	Yarn count		Density		Twist direction	
	warp	weft				warp	weft	warp	weft	warp	weft
Cotton	BD	BD	0,44	158,6	plain	Nm 34	Nm 34	27	22	Z	S

Table 1. Properties of measured fabric sample

The mechanical properties of the examined fabric were determined with a Kawabata Evaluation System (KES) at the Department of Textile Materials and Design at the University of Maribor. The obtained tensile testing diagram is shown in Figure 8, while the bending diagram can be seen in Figure 9. The KES system is universally used and accepted equipment, hence the values obtained from it provide a good basis for comparison.

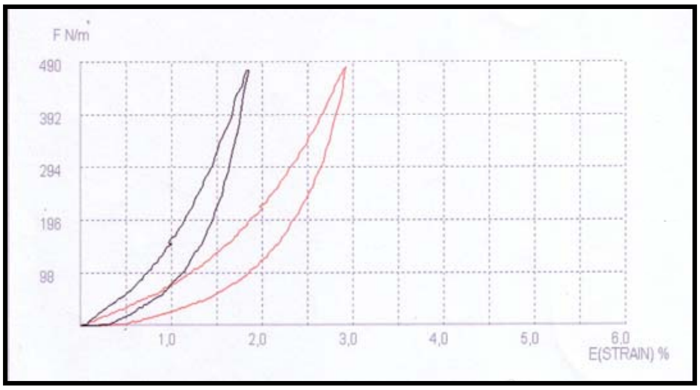

Figure 8. Tensile test

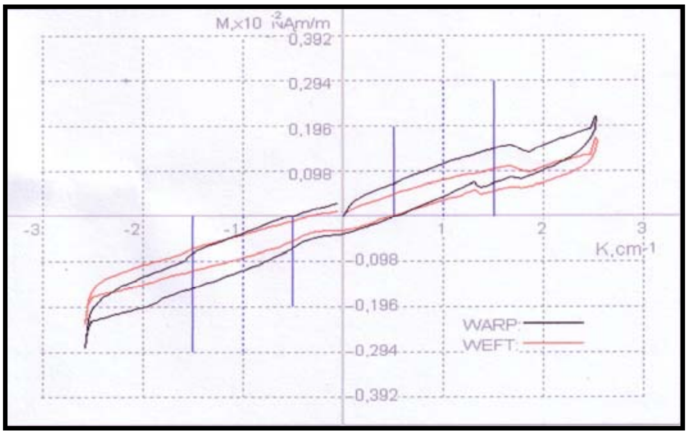

Figure 9. Bending test

The cross sectional inertial moment can be calculated based on the geometry of the examined sample. Using this data and the values measured with the KES system on the 200 mm wide samples gripped at 50 mm span length, the tensile and bending modulus of the fabric can be calculated. The results are summarized in Table 2.

The points obtained with image processing from the photos of the deflecting textile are shown in Figure 10. Polynomials of different order are fitted onto the measurement points graphed in an x-y coordinate system. In case of the polynomials of fourth and sixth power correct behavior (touching horizontal level) could be prescribed, while in case of the second power – due to the low number of parameters – this is not possible. As Figure 10 reveals, a simple parabola of second power according to (15) fits to the line of points correspondent to the measured small deflection ratio (7…8)mm/300mm=0.023…0.027 well – except for the curvature at the ends; hence it is used together with equation (11) in order to determine the mechanical parameters according to chapter 5. There are no other significant points on the parabola – except for the minimum point – that can enhance calculation such as an inflection point or an end point with zero steepness. For this reason only information in the vicinity of the zero point was used in the simplified calculations, the aim of which were only demonstration, to determine bending stiffness, while the arising tensile force and average strain are estimated as free parameters. Table 2 includes the results obtained with the parabola of second order and the estimation of tensile force.

$$y = 0.000359x^2 - 0.00153x, \qquad R^2 = 0.9976 \tag{15}$$

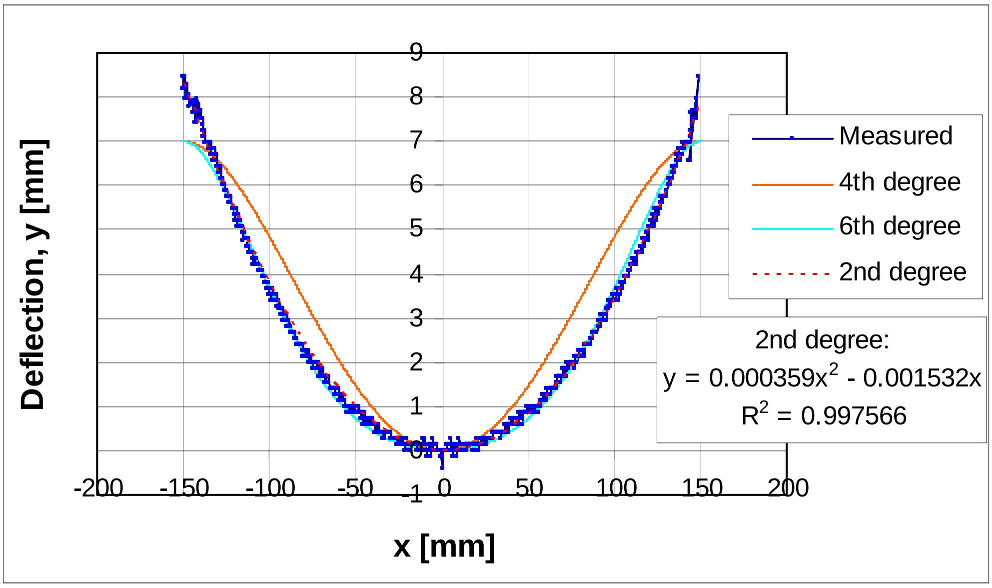

Figure 10. Values measured with the Bending Tester and the fitted curve

		KES System	Bending Tester
Tensile force	**N_0 [N]**	-	0.21
Young modulus	**E [MPa]**	18.18	19.54
Bending modulus	**E' [MPa]**	0.817	0.820

Table 2. Results obtained with the KES system and the Bending tester

It can be concluded based on Table 2 and Figure 8 that the small value of fabric tensile force (0.21 N) – which is the resultant of the weight of the fabric band hanging down and the tension and loosening at the grip of the material that cannot be avoided – is 2.1 N/m projected on 100 mm length. This value falls in the immediate vicinity of the zero point that can be approximated with linearly flexible behavior.

The evaluation method is improved based on the application of polynomials of higher power; hence prescriptions for the several free parameters are to be used in a way that the real, usable data are provided without estimation. Therefore, the fact that the results obtained with the new measurement and evaluation method correspond well with the values determined with the KES system demonstrates that the new equipment and evaluation method will be applicable in the examination of the bending properties of textiles.

5 Summary

A contactless optical measuring method was developed for determining the bending properties among them the bending stiffness of textiles such as woven or knitted fabrics, fleeces, and mats. The shape of specimen deformed under its own weight is scanned by laser beams and recorded by a camera. Using suitable calibration the surface can be described by polynomial regressions that can be treated as the solution of a differential equation derived for the bent sample. After substituting the measured solution into the differential equation linear algebraic equations can be obtained from which the bending parameters can be estimated. The presented results compared to the values obtained with KES measurements demonstrate the applicability of the new measurement method that is still being developed.

Acknowledgement

This work has been supported by the National Office for Research and Technology (NKTH) and by the Agency for Research Fund Management and Research Exploitation (OTKA) of the Hungarian Government with the K 68438 OTKA – NKTH and with projects TR-17/2008, HR-37/2008 and SI-6/2007 S&T.

The authors would like to express their thanks to Prof. Dr.-Sc. Jelka Gersak for the possibility of performing measurements on the KES system at the University of Maribor.

References

[1] Peirce F. T.: The "handle" of cloth as a measurable quantity, The Journal of the Textile Institute **21**, 377-416 (1930).

[2] Kawabata S. et al: Application of Objective Measurement to Clothing Manufacture, International Journal of Clothing Science and Technology **48**, 2, pp. 18-25 (1992).

[3] Tamás P.; Geršak J.; Halász M.: Sylvie 3D Drape Tester – New System for Measuring Fabric Drape, TEKSTIL, Zagreb, 2006/10, P 497-502, ISSN 0492-5882 (2006).
[4] Szirmay-Kalos L., Antal Gy., Csonka F.: “Computer Graphics” Computer Books, Budapest. (2003).
[5] Kim D.K., Jang B.T., Hwang C.J.:”A Planar Perspective Image Matching using Point Correspondence and Rectangle-to-Quadrilateral Mapping” Fifth IEE Southwest Symposium on Image Analysis and Interpretation (2002).
[6] Tamás, P., Halász, M., Gräff, J (2005): “3D Dress Design“ AUTEX World Textile Conference, Portorož, Slovenia. pp 436-440
[7] Williams J.G.: Stress Analysis of Polymers. Longman Group Ltd. London, (1973).
[8] Ponomarjov Sz. D.: Strength Calculations in the Mechanical Engineering. Vol. **2**. Beams, Springs. (in Hungarian). Műszaki Könyvkiadó, Budapest. (1964).
[9] Muttnyánszky Á.: Szilárdságtan/Statics (in Hungarian) Műszaki Könyvkiadó, Budapest, (1981).
[10] Kaliszky S.: Mechanics II. Strength of Materials. (in Hungarian) Tankönyvkiadó, Budapest, (1990).