

Modeling the Deformation and the Failure Process of Glass Woven Fabrics Based on  
the Fibre-Bundle-Cells Theory

Virag Á. D., Vas L. M., Molnár K.

Accepted for publication in Acta Materialia Transylvanica

Published in 2021

DOI: <https://doi.org/10.33924/amt-2021-01-10>

# Modeling the Deformation and the Failure Process of Glass Woven Fabrics Based on the Fibre-Bundle-Cells Theory

Ábris Dávid VIRÁG,<sup>1</sup> László Mihály VAS,<sup>2</sup> Kolos MOLNÁR<sup>3</sup>

<sup>1</sup> *Budapest University of Technology and Economics, Faculty of Mechanical Engineering, Department of Polymer Engineering, Budapest, Hungary, viraga@pt.bme.hu*

<sup>2</sup> *Budapest University of Technology and Economics, Faculty of Mechanical Engineering, Department of Polymer Engineering, Budapest, Hungary, vas@pt.bme.hu*

<sup>3</sup> *Budapest University of Technology and Economics, Faculty of Mechanical Engineering, Department of Polymer Engineering, Budapest, Hungary and MTA–BME Research Group for Composite Science and Technology, Budapest, Hungary, molnar@pt.bme.hu*

---

## Abstract

In this study, we modeled the deformation and failure behavior of different glass woven fabrics under uni-axial tension using the Fibre Bundle Cells-modeling method. The difference between the analytical, phenomenological model curve and the mean curve calculated from the measurement results was classified by the relative mean squared error (RMSE), which is closely related to the coefficient of determination. This value was less than 3.6% in all the examined cases, which indicated good modeling.

**Keywords:** *FBC modeling, glass woven, tensile test.*

---

## 1. Introduction

Nowadays, the application fields of polymer composites are constantly expanding, and in addition to fiber-reinforced polymer composites [1] research on nano- [2], and biocomposites [3], as well as on the destruction of composite structures [4], is gaining prominence. The high use of polymer composites is well illustrated by the fact that more than 50% of the raw materials used in aircraft manufacturing are polymer composites (excluding engines) [5].

A significant part of these are fabric-reinforced composites, hence there is a huge demand for models which can properly describe the mechanical and failure behavior of fabrics and fabric-reinforced composite materials and products already in the design phase. However, this requires fabric models that allow a sufficiently accurate description of the mechanical behavior of the fabrics. Continuous or discrete element models are basically used to model the behavior of woven fabrics.

### 1.1. Continuum models

Most woven fabric models are continuum models that treat fabric as a continuum material. Traditionally, layer models of composite mechanics are used to model fabrics. In order to describe the fabric-reinforced composites more accurately, several analytical and finite element 2D and 3D models have been developed that already take into account the fibrous structure of the fabrics and the structure resulting from the weaving technology. Examples are the mosaic model developed by Ishikawa [6], and the fiber undulation model [7]. The mosaic model first divides the fabric into two layers perpendicular to each other and then separates the warp and weft yarns within the layers. The fiber undulation model takes into account the continuity, the unevenness of the yarns and the intersection of warp and weft yarns as well. It describes the fabric with a repeating unit consisting of three parts: a two-layer element where the two layers are oriented perpendicularly (which is also used at the yarn intersections of the mosaic model), a corrugated element (which describes the

actual intersection of warp and weft yarns), and an element assuming pure matrix material. Later, 2D and 3D modeling have developed from these basic models, and the models applied today can be used to describe time- and temperature-dependent behaviors as well [8].

## 1.2. Discrete element models

Discrete element models define fabric as a network of a finite number of nodes (typically mass points), and nodes are usually connected by mechanical elements with specific properties (e.g., springs, dashpots, etc.). One such model is based on a network of springs and dashpots connected in parallel by Gräff and Kuzmina [9]. In the model, discrete points of mass are connected by different springs (structural, bending and shear). However, these are not simple springs, but essentially Kelvin-Voigt elements (since the spring branches also contain dashpots). The model considers the springs to have a linear characteristic and the dashpots to be proportional to the speed, while not taking into account the friction between the yarns.

Another discrete element model suitable for describing woven fabrics is the Vas's fiber bundle cell model [10, 11]. The Fiber Bundle Cell-based modeling method offers a set of model elements that can be used to build a system for describing the behavior of fibrous structures and composites reinforced with fibrous structures. The Fiber Bundle Cell model is a discrete element model that takes into account that the fabric used as a composite reinforcement is not a continuum structure. The model also considers that the yarns (as fibers) that constitute the fabric form fiber bundles (fiber bundle cells) at the microscopic level. When describing fabrics, the model considers the warp and weft yarns of the fabric as fiber bundles. At the macroscopic level, it describes woven fabrics as a concentrated parameter network of mechanical elements (fiber bundle cells) with discrete, statistical, linear, or nonlinear characteristics. The model assumes that the fibers in a partially ordered fiber bundle of the same type of fibers can be classified based on their initial state and environmental properties (gripping conditions) and can be shifted along with their environment (or into a similar environment). Fibers in the same classes form a sub-fiber bundle (i.e. a fiber bundle cell). The system of fiber bundle cells produced in this way, parallel to the direction of stretching, models the structure and strength of the original bundle.

## 2. The applied model

In the model, the real fabric of warp and weft yarns was replaced by a model with only warp yarns. This was possible because during the warp direction strip tensile tests, the weft yarns essentially do not take up the load, only modify the spatial arrangement and deformation behavior of the warp yarns. Therefore, a model bundle is required whose elements are virtual warp yarns that also include the effect of weft yarns. For  $0^\circ$  specimens, "strong" model yarns are obtained. This step can also be done for  $45^\circ$  specimens, but in this case there is no yarn in the direction of the load. Therefore, as a result of waviness and obliquity modifying the deformation behavior, "weak" (warp) model yarns are obtained.

### 2.1. Modeling with one nonlinear E-bundle

A nonlinear E-bundle from the model set was used to model the tensile tests on the  $0^\circ$  fabric bands. The E-bundle is a well-arranged, elastic, breaking bundle with independent fibers, ideal grips at both ends (i.e., the fibers do not slip out of the grip and do not break in the grip), the fibers are parallel to each other and to the direction of pulling and are not pre-stressed (i.e. they are stress-free but not loose). In this case, the tensile characteristic ( $f(\varepsilon)$  ( $\text{N}\cdot\text{g}^{-1}\cdot\text{m}^2$ )) is described by Equation (1), where  $\varepsilon$  denotes the relative elongation,  $a > 0$  ( $\text{N}\cdot\text{g}^{-1}\cdot\text{m}^2$ ),  $b > 0$  ( $\text{N}\cdot\text{g}^{-1}\cdot\text{m}^2$ ) are the parameters representing the influence of the waviness of the yarn and the wrinkling of the woven strip on the tensile characteristic, and  $c > 0$  ( $\text{N}\cdot\text{g}^{-1}\cdot\text{m}^2$ ) is the asymptotic tensile stiffness:

$$f(\varepsilon) = c\varepsilon + a(1 - e^{-b\varepsilon}) \quad (1)$$

It can be seen that for  $a = 0$  and/or  $b = 0$ , Equation (1) also includes the linear case. However, Equation (1), like the tensile characteristics in general, only describes the intact operation, which can be determined for most mechanical models. Fiber bundle cell-based modeling, on the other hand, assumes that there are defects in the test material. These are considered with a so-called reliability function ( $g(\varepsilon)$ ) (2), where  $\varepsilon$  is the specific elongation of the model yarn,  $\varepsilon_S$  is the specific elongation at break of the model yarn, and  $Q_{\varepsilon_S}$  is the distribution function of the specific elongation at break:

$$g(\varepsilon) = (1 - Q_{\varepsilon_S}(\varepsilon)) \quad (2)$$

In this case, the reliability function is a complementary distribution function of the specific elongation at break. The reliability function gives

the ratio of the still operating, i.e. intact fibers at the given load level ( $\varepsilon$ ). In the present case, we assumed the specific elongation at break ( $\varepsilon_b$ ) of the model yarns to be normally distributed. Using equations (1) and (2), the tensile stress  $\sigma(\varepsilon)$  can be produced as the product of the tensile characteristic  $f(\varepsilon)$  and the reliability function  $g(\varepsilon)$  (3), which significantly reduces the operation of the calculations for more complex calculations:

$$\sigma(\varepsilon) = f(\varepsilon) \cdot g(\varepsilon) \quad (3)$$

## 2.2. Modeling with two nonlinear E-bundles connected in parallel

Two parallel-connected nonlinear E-bundles were used in each case to model the mean curves describing the deformation and failure behavior of 45 ° angled woven strips under uniaxial tension. The use of the two bundles was necessary due to the shape of the curves, as a very inaccurate result would have been obtained with a single bundle model. The two-bundle case is very similar to the single bundle case, however, the tensile characteristic is described not by Equation (1) but by Equation (4), where  $p_1$  and  $p_2$  are weights for which it is true that  $p_1 + p_2 = 1$ :

$$f(\varepsilon) = p_1 \cdot f_1(\varepsilon) + p_2 \cdot f_2(\varepsilon) \quad (4)$$

where the characteristic of each bundle is (5):

$$f_i(\varepsilon) = \begin{cases} c_i \cdot (\varepsilon - d_i) + a_i(1 - e^{-b_i(\varepsilon - d_i)}), & \text{if } \varepsilon \geq d_i \\ 0, & \text{if } \varepsilon < d_i \end{cases} \quad (5)$$

where  $i \in \{1, 2\}$ , } and  $d$  is an offset that is zero in one or both cases. It can be seen that Equation (5) also includes the single bundle case, where  $d = 0$ . In the two-bundle case, for example, for one bundle  $d > 0$ , and for the other  $d = 0$ . Thus, in the two-bundle case, the tensile characteristic has four parameters. In this case the specific tensile stress also can be obtained as the weighted sum of the product of the tensile characteristic and the reliability function on the two nonlinear bundles  $E(E_1$  and  $E_2)$  :

$$\begin{aligned} \sigma(\varepsilon) &= \sigma_{E_1}(\varepsilon) + \sigma_{E_2}(\varepsilon) \\ &= p_1 \cdot \sigma_1(\varepsilon) + p_2 \cdot \sigma_2(\varepsilon) \\ &= p_1 \cdot f_1(\varepsilon) \cdot g_1(\varepsilon) + p_2 \cdot f_2(\varepsilon) \cdot g_2(\varepsilon) \end{aligned} \quad (6)$$

## 3. Materials and methods

In the research, UTE195P from the manufacturer Unique Textiles (area density: 195 g/m<sup>2</sup>, weave: plain) and UTE195T (area density: 195 g/m<sup>2</sup>, weave: twill) glass fabrics were tested. Warp-directional and  $\pm 45^\circ$  samples were prepared from the woven fabrics, with a width and length of 50 and 200 mm, respectively. Uniaxial tensile testing

of the samples was performed on a universal tensile machine (manufacturer: Zwick, type: Z005). The grip length was 100 mm and the test speed was 25 mm/min, during the test, the force and the crosshead displacement were recorded. FBC modeling was performed based on tensile tests. The five force-crosshead displacement curves were first parameterized to the specific stress (ratio of the measured force and the area density of the specimen) – specific elongation (ratio of the actual and initial clamping length of the specimen) curve, and then determined in each case by the so-called smoothed mean curve (this was obtained by moving average smoothing, where the width of the smoothing window was  $\varepsilon = 0.01$ ). The resulting mean curves were used for modeling. Using Equations (1) - (6), we performed the FBC modeling for the total specific stress -specific elongation curve. The model parameters ( $a_p$ ,  $b_p$ ,  $c_p$ ,  $d_p$  and  $p_i$ ) in Equations (1) and (5) were optimized by an iteration method, so that the difference between the model curve and the mean curve should be optimized to get a minimum RMSE value (Eq. 7), which is closely related to the coefficient of determination:

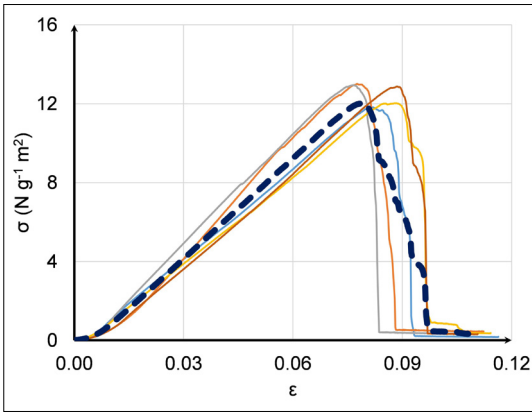
$$RMSE = \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n (\sigma_{measured}(\varepsilon_i) - \sigma(\varepsilon_i))^2} \quad (7)$$

where  $n$  is the number of measuring points,  $\sigma_{measured}$  is the specific stress calculated from the measured force values, and  $\varepsilon_i$  is the elongation value corresponding to the stress measured at the given point.

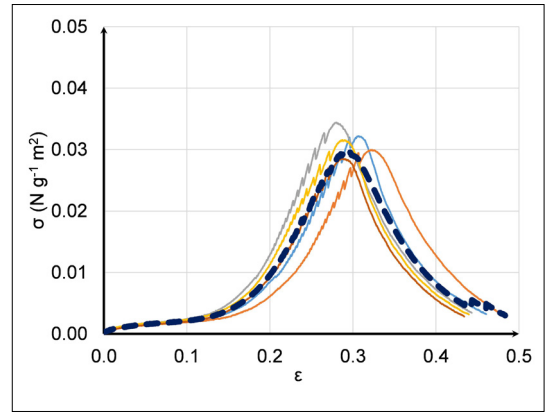
## 4. Results and discussion

The specific stress ( $\sigma$ )-specific elongation ( $\varepsilon$ ) curves calculated from the tensile tests for the tested woven fabrics are shown in [Figures 1-4](#). In the figures, the dashed line indicates the smoothed mean curve required for the modeling.

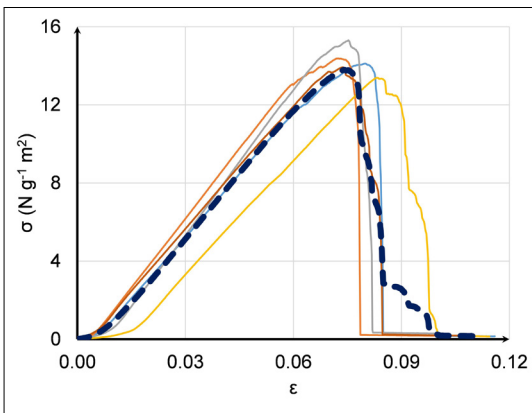
The curves show that there is no significant difference between the force maxima of the test specimens of the same size but with different weaves in the warp direction. However, in the case of  $\pm 45$  degrees, there is an order of magnitude difference. Since there is no yarn in the direction of the load in this test, the maximum force is determined by the weave of the fabrics, since all their other parameters are the same. These measurement results also prove that twill weaving results in a much looser structure, which also implies that it is much easier to transform into



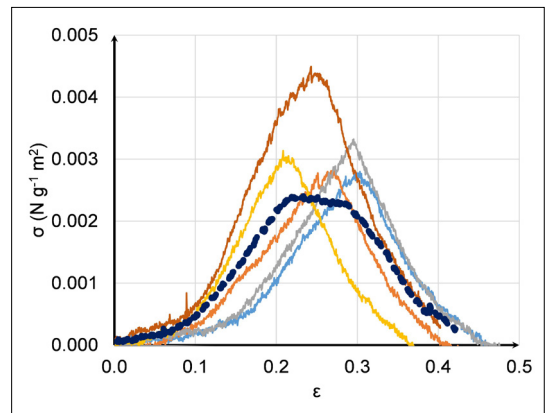
**Figure 1.** Warp directional tensile curves and the smoothed mean curve (dashed line) of plain fabric.



**Figure 3.**  $\pm 45^\circ$  directional tensile curves and the smoothed mean curve (dashed line) of plain fabric.



**Figure 2.** Warp directional tensile curves and the smoothed mean curve (dashed line) of twill fabric.



**Figure 4.**  $\pm 45^\circ$  directional tensile curves and the smoothed mean curve (dashed line) of twill fabric.

complex 3D shapes. Of course, this difference also occurs in the mechanical properties when used as an embedded reinforcement. The relationship between the smoothed mean curves determined from each measurement curve and the modeled curves is shown in **Figures 5–8**.

The quantified results of the modeling are shown in **Table 1**. In the table, parameters  $a$ ,  $b$ ,  $c$ ,  $d$  are the variables in Equations (1) and (5), and  $E(\varepsilon_s)$ ,  $D(\varepsilon_s)$  and  $V(\varepsilon_s)$  are the expected value, standard deviation, and relative standard deviation, respectively. To characterize the relationship between the model curves and the smoothed mean curves, i.e. to determine the goodness of the modeling, we used the relative mean squared error (RMSE).

It can be seen from **Table 1**, that the relative mean squared error between the smoothed mean

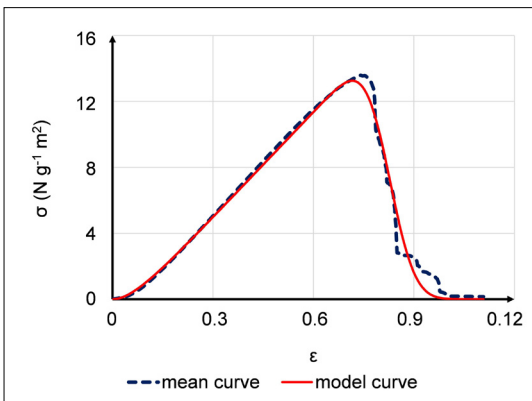
curve and the curve calculated by the Fiber Bundle Cell theory is less than 3.6% in every case. Based on these results, the modeling can be considered exceptionally good, as the modeled curve approximates the smoothed mean curve well enough.

## 5. Conclusions

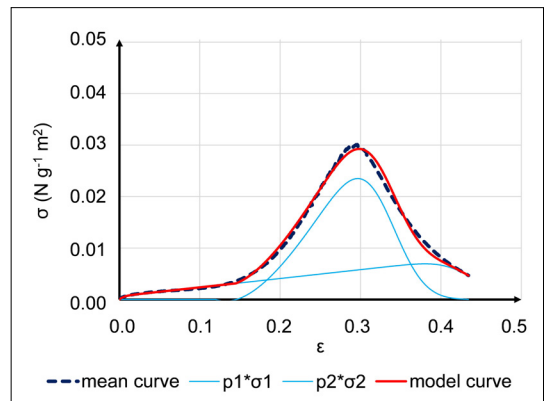
Using the Fiber Bundle Cell theory, we created a discrete element, analytical, phenomenological model suitable for the mechanical modeling of the deformation and failure behavior of the investigated composite reinforcing fabrics during uniaxial tensile testing. The relative mean squared error between the smoothed mean curve determined from 5 tensile tests per material and the curve calculated from the model was less than 3.6% in each case.

**Table 1.** Parameters characterizing the modeling of woven fabric strips

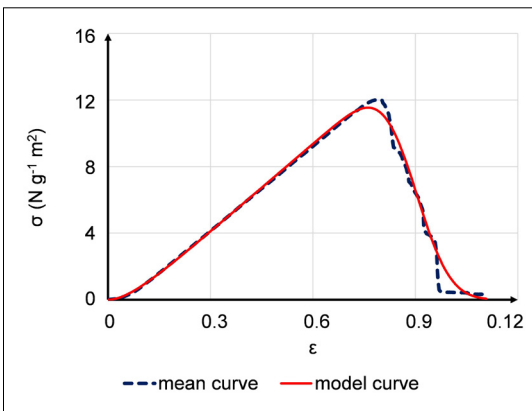
Sample and grip			MODEL PARAMETERS								
			Model-structure	Tensile characteristics - intact functioning				Failure			Fitting
Sam-ple	Grip distance (mm)	Azi-muth angle (°)	Com-ponent weight, $p$	$a$ ( $N \cdot g^{-1} \cdot m^2$ )	$b$ ( $N \cdot g^{-1} \cdot m^2$ )	$c$ ( $N \cdot g^{-1} \cdot m^2$ )	$d$	$E(\epsilon_s)$	$D(\epsilon_s)$	$V(\epsilon_s)$ (%)	RMSE (%)
Plain	100	0	1.00	-220	1.5	330	0	0.09	0.008	9.0	3.05
Twill	100	0	1.00	-267	1.5	400	0	0.08	0.006	7.1	3.49
Plain	100	45	0.40	0.3	2.0	0.08	0	0.44	0.033	7.5	2.10
			0.60	-8.3	0.1	0.92	12	0.33	0.033	10.0	
Twill	100	45	0.12	0.3	0.5	0.01	0	0.34	0.063	18.5	3.59
			0.88	-0.6	0.1	0.06	6	0.29	0.063	21.7	



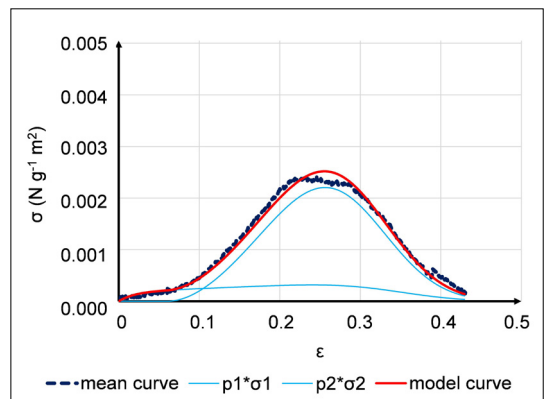
**Figure 6.** Warp directional mean curve and model curve of twill fabric obtained from tensile tests.



**Figure 7.**  $\pm 45^\circ$  directional mean curve and model curve of plain fabric obtained from tensile tests.



**Figure 5.** Warp directional mean curve and model curve of plain fabric obtained from tensile tests.



**Figure 8.**  $\pm 45^\circ$  directional mean curve and model curve of twill fabric obtained from tensile tests.

Based on the results, the modeling method is proven to be exceptionally good. It can be stated that the FBC modeling is a fast, simple method with a small computing capacity, which is suitable for describing the deformation and failure behavior of fabrics under uniaxial tension.

### Acknowledgements

This research was supported by the ÚNKP-20-3 and ÚNKP-20-5 New National Excellence Program of the Ministry for Innovation and Technology. The research reported in this paper and carried out at BME has been supported by the NRDIFund (TKP2020 NC, Grant No. BME-NC) based on the charter of bolster issued by the NRDIFund under the auspices of the Ministry for Innovation and Technology.

### References

- [1] Rajak D. K., Pagar D. D., Menezes P. L., Linul E.: *Fiber-Reinforced Polymer Composites: Manufacturing, Properties, and Applications*. Polymers, 11/10. (2019) 1667.  
<https://doi.org/10.3390/polym11101667>
- [2] Singh R., Shrivastava A. K., Bajpai A. K.: *CdSe reinforced polythiophene nanocomposites as excellent materials for diode applications*. eXPRESS Polymer Letters, 15/1. (2021) 45–57.  
<https://doi.org/10.3144/expresspolymlett.2021.6>
- [3] Akintayo O. S., Olajide J. L., Betiku O. T., Egho A. J., Adegbesan O. O., Daramola O. O., Sadiku E. R., Desai D. A.: *Poly(lactic acid)-silkworm silk fibre/fibroin bio-composites: A review of their processing, properties, and nascent applications*. eXPRESS Polymer Letters, 14/10. (2020) 924–951.  
<https://doi.org/10.3144/expresspolymlett.2020.76>
- [4] Takács L., Szabó F.: *Experimental and Numerical Failure Analysis of Adhesive Joint of Glass Fiber Reinforced Polymer Composite*. Periodica Polytechnica Mechanical Engineering, 64/1. (2020) 88–95.  
<https://doi.org/10.3311/PPme.15106>
- [5] 1001Crash.com: *Worries about new composite made airplane*, 2011. (letöltve: 2021. február 17.).  
<http://www.1001crash.com/index-page-composite-lg-2.html>
- [6] Ishikawa T.: *Anti-symmetric Elastic Properties of Composite Plates of Satin Weave Cloth*. Fiber Science and Technology, 15/2. (1981) 127–145.  
[https://doi.org/10.1016/0015-0568\(81\)90066-X](https://doi.org/10.1016/0015-0568(81)90066-X)
- [7] Ishikawa T., Chou T. W.: *One-dimensional Micro-mechanical Analysis of Woven Fabric Composites*. AIAA, 21/12. (1983) 1714–1721.  
<https://doi.org/10.2514/3.8314>
- [8] Shrotriya P., Sottos N. R.: *Viscoelastic Response of Woven Composite Substrates*. Composites Science and Technology, 65/3–4. (2005) 621–634.  
<https://doi.org/10.1016/j.compscitech.2004.09.002>
- [9] Gräff J., Kuzmina J.: *Cloth Simulation Using Mass and Spring Model*. In: Material Science. Mechanical Engineering 2004. Proceeding of 4<sup>th</sup> Conference on Mechanical Engineering, Budapest, Hungary, Budapest, BUTE, (2004) 443–447.
- [10] Vas L. M.: *Idealizált statisztikus szálkötegcellák és alkalmazásuk szálas szerkezetek, kompozitok modellezésére*. MTA doktori értekezés (2007).
- [11] Vas L. M., Tamás P.: *Modelling Method Based on Idealised Fibre Bundles*. Plastics, Rubber and Composites, 37/5–6. (2008) 233–239.  
<https://doi.org/10.1179/174328908X309411>